AJAE appendix for "Modeling Farm Households' Price Responses in the Presence of Transaction Costs and Heterogeneity in Labor Markets"

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Contents

1 Motivation of the Labor Market Model

1.1 Simultaneous Demand of On-Farm Labor and Supply of Off-Farm Labor

Simultaneously demanding on-farm labor and supplying off-farm labor can be rational with a strictly convex labor cost function and a strictly concave labor income function. To observe this, assume that in autarky the shadow price of labor on the farm would be lower than the marginal revenue of selling off-farm labor and higher than the marginal cost of hiring on-farm labor. Obviously, under this assumption, utility maximizing implies that the farm household supplies off-farm labor until marginal revenue equals the shadow price of labor, while the household demands on-farm labor until marginal cost equals the shadow price of labor or hired labor equals optimal labor input, i.e. the household no longer works on its own farm. Now, given strict convex and strict concave labor costs and income functions, there always exists an interior solution, i.e. the household simultaneously supplies and demands labor and works on its own farm. For instance, if the skills of the household members to work off-farm are very heterogeneous, it is rational to simultaneously supply high-priced labor of well-educated household members and hire cheap agricultural labor (see also Sadoulet, de Janvry, and Benjamin 1996).

1.2 Examples of Non-proportional Variable Transaction Costs

In this section we provide some intuitive examples of non-proportional variable transaction costs (NTC). It is well recognized in the literature that participation in rural labor markets is often plagued by adverse selection and moral hazard problems due to asymmetric information regarding the quality of the labor force (Eswaran and Kotwal 1986; Spence 1976) and the effort of hired labor, respectively (Frisvold 1994; Sadoulet, de Janvry, and Benjamin 1998; Eswaran and Kotwal 1986). Generally, moral hazard and adverse selection problems might change non-proportionally with the quantity of traded goods, implying NTC for both on-farm labor demand and off-farm labor supply. Theoretically, it is unclear how these costs vary, i.e. if they are increasing, decreasing, or proportional to the amount of hired or supplied labor.

For example, in the case of moral hazard problems of hired on-farm labor, it is well recognized that employers cannot easily infer labor effort indirectly by observing final output, due to the stochastic and seasonal nature of agricultural production. Therefore, supervision costs rise to control for moral hazard problems (Frisvold 1994; Feder 1985). Marginal costs to supervise hired labor may increase along with the units of hired labor due to an increase in the probability of free-riding, the greater importance of coordinating work inputs, and the increased effort to control for social conflicts among employees.

Moreover, adverse selection problems due to asymmetric information on the quality of hired labor might lead to transaction costs in rural labor markets. These transaction costs might be partially reduced by adequate formal institutions (Spence 1976). However, in rural labor markets, adequate formal institutions that avoid adverse selection problems, e.g. formal education certificates, are often incompletely developed. In that case, a firm might use informal screening mechanisms to learn about the quality of workers, e.g. information from peer groups or rural organizations (Granovetter 1973; Sadoulet, de Janvry, and Benjamin 1998). Accessability to peer groups or rural organizations varies, i.e. workers living in the neighborhood might have more access than those living in a more distant village. Thus, the potential to control for adverse selection problems increases when firms shift their demand from local to regional labor markets, implying increasing marginal NTC.

Moreover, even if information, search, and bargaining costs are considered as fixed costs, they occur for each labor contract. Therefore, from the perspective of the farm household, total costs, including all labor contracts, are no longer fixed costs but vary with the number of workers. Finally, other transaction costs might also vary with the number of labor contracts, e.g. there are only slight additional costs if one or two people travel to the city in the same car or family members who work for the same firm might reduce search and bargaining costs for succeeding family workers. On the other hand, some part-time jobs might be available near the farm, while full-time jobs are only available in larger settlements farther away, implying increasing transportation costs.

2 Theoretical Results

Table 1. Theoretical Effects of Exogenous Price Changes

Behavior	Variable	Non-separable Model				Separable Model				
		P_c	P_a	P_v	P_m	P_c	P_a	P_v	P_L	P_m
Farm	X_c	?	?	?	?	+	?	(-)	(-)	0
	X_a	?	?	?	?	?	+	(-)	(-)	0
	$\|X_v\|$	?	?	?	?	(+)	(+)	-	(-)	0
	$\|X_L\|$	?	?	?	?	(+)	(+)	(-)	-	0
Consumption	C_m	(+)	(+)	(-)	?	(+)	(+)	(-)	(+)	(-)
	C_a	(+)	?	(-)	?	(+)	?	(-)	(+)	?
	C_L	?	?	?	?	(+)	(+)	(-)	?	?
Labor market	X_L^n	(-)	(-)	(+)	?	(-)	(-)	(+)	(+)	?
	X_L^s	(-)	(-)	(+)	?					
	X_L^h	(+)	(+)	(-)	?					
	P_L^*	(+)	(+)	(-)	?					

Note: It is assumed that goods are not inferior, technologies are not regressive, and households are net suppliers of labor and self-produced agricultural goods.

Variables: $X_.$ = netput quantities, $C_.$ = consumed quantities, $P_.$ = exogenous prices, $P_.^*$ = endogenous shadow prices; subscripts: c = crop products, a = animal products, v = variable inputs, L = labor/leisure; superscripts of X_L (labor quantities): h = hired, s = supplied, n = net supplied.

Symbols indication the direction of the effects:

0	=	clear, no effect;
+/-	=	clear, increase/decrease;
(+)/(-)	=	unclear, but most likely an increase/decrease (assuming labor and variable inputs are complements, and consumption goods are net-substitutes);
?	=	unclear.

3 Symmetric Normalized Quadratic (SNQ) Profit Function

This functional form is also traded under the name of "symmetric generalized McFadden function" (Diewert and Wales 1992).

3.1 First Stage Profit Function

We follow Lopez (1984) and determine the shadow price of labor on the farm by estimating a profit function assuming constant returns to labor. In this case a symmetric normalized quadratic (SNQ) profit function (Diewert and Wales 1987, 1992; Kohli 1993) has following form:

$$\text{(1)} \quad \Pi\left(\boldsymbol{p}_{pn}, \boldsymbol{r}_n, X_{Ln}\right) = X_{Ln} \left(\begin{array}{l} \sum_{i\in\{c,a,v\}} \alpha_i P_{in} + \frac{1}{2} w_n^{-1} \sum_{i\in\{c,a,v\}} \sum_{j\in\{c,a,v\}} \beta_{ij} P_{in} P_{jn} \\ + \sum_{i\in\{c,a,v\}} \sum_{j\in\{g,k\}} \delta_{ij} P_{in} R_{jn} + \frac{1}{2} w_n \sum_{i\in\{g,k\}} \sum_{j\in\{g,k\}} \gamma_{ij} R_{in} R_{jn} \end{array} \right)$$

where n indicates the observation (household), Π is the profit function, X_{Ln} is the labor deployed on the farm, $w_n = \sum_{i\in\{c,a,v\}} \theta_i P_{in}$ is a factor to normalize prices, $\theta_i = \sum_n P_{in} |X_{in}| / \sum_n \sum_{j\in\{c,a,v\}} P_{jn} |X_{jn}|$; $i \in \{c,a,v\}$ are predetermined weights of the individual netput prices, $\boldsymbol{p}_{pn} = (P_{an}, P_{cn}, P_{vn})$ indicates the netput prices, X_{in}; $i \in \{c,a,v\}$ denotes the quantity indices of the netputs, $\boldsymbol{r}_n = (R_{gn}, R_{kn})$ represents the quasi-fixed factors land (R_g) and capital (R_k), and α_i, β_{ij}, δ_{ij}, and γ_{ij} are the parameters to be estimated. To identify all β_{ij}, we impose the restrictions $\sum_{j\in\{c,a,v\}} \beta_{ij} \overline{P_j} = 0$; $i \in \{c,a,v\}$, where $\overline{P_j}$ are the mean prices (Diewert and Wales 1987, p. 54).

The corresponding netput equations can be obtained using Hotelling's Lemma:

$$\text{(2)} \quad X_{in}\left(\boldsymbol{p}_{pn}, \boldsymbol{r}_n, X_{Ln}\right) = \frac{\partial \Pi\left(\boldsymbol{p}_{pn}, \boldsymbol{r}_n, X_{Ln}\right)}{\partial P_{in}}$$

$$\text{(3)} \quad = X_{Ln} \left(\begin{array}{l} \alpha_i + w_n^{-1} \sum_{j\in\{c,a,v\}} \beta_{ij} P_{jn} \\ -\frac{1}{2} \theta_i w_n^{-2} \sum_{j\in\{c,a,v\}} \sum_{k\in\{c,a,v\}} \beta_{jk} P_{jn} P_{kn} \\ + \sum_{j\in\{g,k\}} \delta_{ij} R_{jn} + \frac{1}{2} \theta_i \sum_{j\in\{g,k\}} \sum_{k\in\{g,k\}} \gamma_{jk} R_{jn} R_{kn} \end{array} \right)$$

3.2 Second Stage Profit Function

At the second stage we estimate a symmetric normalized quadratic (SNQ) profit function (Diewert and Wales 1987, 1992; Kohli 1993) with labor as variable input:

$$
\begin{aligned}
(4) \quad \Pi\left(\boldsymbol{p}_{pn}, \boldsymbol{r}_n\right) &= \sum_{i\in\{c,a,v,L\}} \alpha_i P_{in} + \frac{1}{2} w_n^{-1} \sum_{i\in\{c,a,v,L\}} \sum_{j\in\{c,a,v,L\}} \beta_{ij} P_{in} P_{jn} \\
&+ \sum_{i\in\{c,a,v,L\}} \sum_{j\in\{g,k\}} \delta_{ij} P_{in} R_{jn} + \frac{1}{2} w_n \sum_{i\in\{g,k\}} \sum_{j\in\{g,k\}} \gamma_{ij} R_{in} R_{jn}
\end{aligned}
$$

where $w_n = \sum_{i\in\{c,a,v,L\}} \theta_i P_{in}$ is a factor to normalize prices, $\theta_i = \sum_n P_{in} \left|X_{in}\right| / \sum_n \sum_{j\in\{c,a,v,L\}} P_{jn} \left|X_{jn}\right|$; $i \in \{c,a,v,L\}$ are predetermined weights of the individual netput prices, $\boldsymbol{p}_{pn} = (P_{an}, P_{cn}, P_{vn}, P_{Ln})$ indicates the netput prices, X_{in}; $i \in \{c,a,v,L\}$ denotes the quantity indices of the netputs, $\boldsymbol{r}_n = (R_{gn}, R_{kn})$ represents the quasi-fixed factors land (R_g) and capital (R_k), and α_i, β_{ij}, δ_{ij}, and γ_{ij} are the parameters to be estimated. To identify all β_{ij}, we impose the restrictions $\sum_{j\in\{c,a,v,L\}} \beta_{ij}\overline{P_j} = 0$; $i \in \{c,a,v,L\}$, where $\overline{P_j}$ are the mean prices (Diewert and Wales 1987, p. 54).

The corresponding netput equations can be obtained using Hotelling's Lemma:

$$
\begin{aligned}
(5) \quad X_{in}\left(\boldsymbol{p}_{pn}, \boldsymbol{r}_n\right) &= \frac{\partial \Pi\left(\boldsymbol{p}_{pn}, \boldsymbol{r}_n\right)}{\partial P_{in}} \\
(6) \quad &= \alpha_i + w_n^{-1} \sum_{j\in\{c,a,v,L\}} \beta_{ij} P_{jn} - \frac{1}{2} \theta_i w_n^{-2} \sum_{j\in\{c,a,v,L\}} \sum_{k\in\{c,a,v,L\}} \beta_{jk} P_{jn} P_{kn} \\
&+ \sum_{j\in\{g,k\}} \delta_{ij} R_{jn} + \frac{1}{2} \theta_i \sum_{j\in\{g,k\}} \sum_{k\in\{g,k\}} \gamma_{jk} R_{jn} R_{kn}
\end{aligned}
$$

4 Labor Market Analysis

4.1 Labor Supply

To estimate the marginal revenue of supplying labor, we assume the following specifications of the average regional wage level $\overline{P}_L$, the household-specific wage shifters b^s, and the variable transaction costs TC_v^s, which include proportional (PTC) and non-proportional variable (NTC) transaction costs:

$$
\begin{aligned}
(7) \quad \overline{P}_L &= \widetilde{P}_L \beta_p^s \\
(8) \quad b^s\left(X_L^s, \boldsymbol{z}_L^s\right) &= \beta_0^s + \boldsymbol{z}_L^{s\prime} \boldsymbol{\beta}_L^s + X_L^s \beta_{L1}^s \\
(9) \quad TC_v^s\left(X_L^s, \boldsymbol{z}_v^s\right) &= \left(\boldsymbol{z}_v^{s\prime} \boldsymbol{\beta}_v^s\right) X_L^s + \beta_{v1}^s X_L^{s2}
\end{aligned}
$$

where $\widetilde{P}_L$ is a proxy for the average regional wage level. The specification of b^s shows that $\boldsymbol{z}_L^{s\prime} \boldsymbol{\beta}_L^s$ indicates general wage differences between the households, while $X_L^s \beta_{L1}^s$ refers to a wage shift due to a changing amount of supplied labor, which is caused by heterogeneity within each household. The specification of TC_v^s is derived from a second-order Taylor series approximation of the true transaction costs (see section 4.3). It shows that $\boldsymbol{z}_v^{s\prime}\boldsymbol{\beta}_v^s$ denotes proportional transaction costs per unit of labor, and $\beta_{v1}^s X_L^{s\,2}$ are non-proportional variable transaction costs.

Substituting these specifications into equation (5) of the main article, we get the empirical specification used for the estimation, which is presented in equation (18) of the main article.

$$\begin{aligned}
(10)\quad P_L^s &= \overline{P}_L + b^s\left(X_L^s, \boldsymbol{z}_L^s\right) - \frac{\partial TC_v^s\left(X_L^s, \boldsymbol{z}_v^s\right)}{\partial X_L^s} \\
(11)\quad &= \widetilde{P}_L\beta_p^s + \beta_0^s + \boldsymbol{z}_L^{s\prime}\boldsymbol{\beta}_L^s + X_L^s\beta_{L1}^s - \boldsymbol{z}_v^{s\prime}\boldsymbol{\beta}_v^s - 2X_L^s\beta_{v1}^s \\
(12)\quad &= \beta_0^s + \widetilde{P}_L\beta_p^s + \boldsymbol{z}_L^{s\prime}\boldsymbol{\beta}_L^s - \boldsymbol{z}_v^{s\prime}\boldsymbol{\beta}_v^s + X_L^s(\beta_{L1}^s - 2\beta_{v1}^s) \\
(13)\quad &= \beta_0^s + \boldsymbol{z}^{s\prime}\boldsymbol{\beta}^s + X_L^s\beta_1^s
\end{aligned}$$

with $\boldsymbol{z}^s = \left(\widetilde{P}_L, \boldsymbol{z}_L^{s\prime}, \boldsymbol{z}_v^{s\prime}\right)'$, $\boldsymbol{\beta}^s = \left(\beta_p^s, \boldsymbol{\beta}_L^{s\prime}, -\boldsymbol{\beta}_v^{s\prime}\right)'$, and $\beta_1^s = \beta_{L1}^s - 2\beta_{v1}^s$.

Neglecting FTC, we can derive the net off-farm labor revenue function f from the estimated coefficients of equation (13) by applying equation (6) of the main article:

$$(14)\quad f\left(X_L^s\right) + TC_f^s = \int_0^{X_L^s}\left(\widehat{\beta}_0^s + \boldsymbol{z}^{s\prime}\widehat{\boldsymbol{\beta}}^s + X_L^s\widehat{\beta}_1^s\right)\mathrm{d}X_L^s = \left(\widehat{\beta}_0^s + \boldsymbol{z}^{s\prime}\widehat{\boldsymbol{\beta}}^s\right)X_L^s + \frac{1}{2}\widehat{\beta}_1^s\, X_L^{s\,2}$$

4.2 Labor Demand

To estimate the marginal cost of hiring labor, we assume the following specifications of the average regional wage level $\overline{P}_L$, the farm-specific wage shifters b^h, and the variable transaction costs TC_v^h, which include PTC and NTC:

$$\begin{aligned}
(15)\quad \overline{P}_L &= \widetilde{P}_L\beta_p^h \\
(16)\quad b^h\left(X_L^h, \boldsymbol{z}_L^h\right) &= \beta_0^h + \boldsymbol{z}_L^{h\prime}\boldsymbol{\beta}_L^h + X_L^h\beta_{L1}^h \\
(17)\quad TC_v^h\left(X_L^h, \boldsymbol{z}_v^h\right) &= \left(\boldsymbol{z}_v^{h\prime}\boldsymbol{\beta}_v^h\right)X_L^h + \beta_{v1}^h X_L^{h\,2}
\end{aligned}$$

where $\widetilde{P}_L$ is a proxy for the average regional wage level. The specification of b^h shows that $\boldsymbol{z}_L^{h\prime}\boldsymbol{\beta}_L^h$ indicates general wage differences between the farms, while $X_L^h\beta_{L1}^h$ refers to a wage shift due to a changing amount of hired labor, which is caused by heterogeneity within the hired workers of each farm. The specification of TC_v^h is derived from a second-order Taylor series approximation of the true transaction costs (see section 4.3). It shows that $\boldsymbol{z}_v^{h\prime}\boldsymbol{\beta}_v^h$ denotes proportional transaction costs per unit of labor, and $\beta_{v1}^h X_L^{h\,2}$ are non-proportional transaction costs.

Substituting these specifications into equation (7) of the main article, we get the empirical specification used for the estimation, which is presented in equation (19) of the main article.

$$
\begin{aligned}
(18)\quad P_L^h &= \overline{P}_L + b^h\left(X_L^h, \boldsymbol{z}_L^h\right) + \frac{\partial TC_v^h\left(X_L^h, \boldsymbol{z}_v^h\right)}{\partial X_L^h} \\
(19)\quad &= \widetilde{P}_L\beta_p^h + \beta_0^h + \boldsymbol{z}_L^{h\prime}\boldsymbol{\beta}_L^h + X_L^h\beta_{L1}^h + \boldsymbol{z}_v^{h\prime}\boldsymbol{\beta}_v^h + 2X_L^h\beta_{v1}^h \\
(20)\quad &= \beta_0^h + \widetilde{P}_L\beta_p^h + \boldsymbol{z}_L^{h\prime}\boldsymbol{\beta}_L^h + \boldsymbol{z}_v^{h\prime}\boldsymbol{\beta}_v^h + X_L^h(\beta_{L1}^h + 2\beta_{v1}^h) \\
(21)\quad &= \beta_0^h + \boldsymbol{z}^{h\prime}\boldsymbol{\beta}^h + X_L^h\beta_1^h
\end{aligned}
$$

with $\boldsymbol{z}^h = \left(\widetilde{P}_L, \boldsymbol{z}_L^{h\prime}, \boldsymbol{z}_v^{h\prime}\right)'$, $\boldsymbol{\beta}^h = \left(\beta_p^h, \boldsymbol{\beta}_L^{h\prime}, \boldsymbol{\beta}_v^{h\prime}\right)'$, and $\beta_1^h = \beta_{L1}^h + 2\beta_{v1}^h$.

Neglecting FTC, we can derive the effective cost function for hired labor g from the estimated coefficients of equation (21) by applying equation (8) of the main article:

$$
(22)\quad g\left(X_L^h\right) - TC_f^h = \int_0^{X_L^h}\left(\widehat{\beta}_0^h + \boldsymbol{z}^{h\prime}\widehat{\boldsymbol{\beta}}^h + \widehat{\beta}_1^h X_L^h\right)\mathrm{d}X_L^h = \left(\widehat{\beta}_0^h + \boldsymbol{z}^{h\prime}\widehat{\boldsymbol{\beta}}^h\right)X_L^h + \frac{1}{2}\widehat{\beta}_1^h\, {X_L^h}^2
$$

4.3 Second-order Taylor Series Approximation of Variable Transaction Costs

We assume that the transactions costs (TC) are a function of the traded quantity (X_L) and some further factors that influence variable transaction costs ($\boldsymbol{z}_v$)[1]:

$$
(23)\quad TC = f\begin{pmatrix} X_L \\ \boldsymbol{z}_v \end{pmatrix}
$$

where X_L is a scalar that represents X_L^s or X_L^h and $\boldsymbol{z}_v$ is a vector that represents $\boldsymbol{z}_v^s$ or $\boldsymbol{z}_v^h$.

[1] In this section we ignore factors that influence fixed transaction costs because we are interested only in variable transaction costs here.

We approximate the true transaction costs at point $\begin{pmatrix} X_L^0 \\ z_v^0 \end{pmatrix}$ by a second-order Taylor series[2]:

$$
\begin{aligned}
(24)\quad TC^* &= f\begin{pmatrix} X_L^0 \\ z_v^0 \end{pmatrix} + \begin{pmatrix} X_L - X_L^0 \\ z_v - z_v^0 \end{pmatrix}' \begin{pmatrix} \frac{\partial TC}{\partial X_L} \\ \frac{\partial TC}{\partial z_v} \end{pmatrix} \\
&+ \begin{pmatrix} X_L - X_L^0 \\ z_v - z_v^0 \end{pmatrix}' \begin{pmatrix} \frac{\partial^2 TC}{\partial X_L^2} & \frac{\partial^2 TC}{\partial X_L \partial z_v} \\ \frac{\partial^2 TC}{\partial X_L \partial z_v} & \frac{\partial^2 TC}{\partial z_v^2} \end{pmatrix} \begin{pmatrix} X_L - X_L^0 \\ z_v - z_v^0 \end{pmatrix} \\
(25)\qquad\quad &= f\begin{pmatrix} X_L^0 \\ z_v^0 \end{pmatrix} + \left(X_L - X_L^0\right)\frac{\partial TC}{\partial X_L} + \left(z_v - z_v^0\right)'\frac{\partial TC}{\partial z_v} + \left(X_L - X_L^0\right)^2 \frac{\partial^2 TC}{\partial X_L^2} \\
&+2\left(X_L - X_L^0\right)\frac{\partial^2 TC}{\partial X_L \partial z_v}\left(z_v - z_v^0\right) + \left(z_v - z_v^0\right)'\frac{\partial^2 TC}{\partial z_v^2}\left(z_v - z_v^0\right) \\
(26)\qquad\quad &= f\begin{pmatrix} X_L^0 \\ z_v^0 \end{pmatrix} + \frac{\partial TC}{\partial X_L}X_L - \frac{\partial TC}{\partial X_L}X_L^0 + \left(z_v - z_v^0\right)'\frac{\partial TC}{\partial z_v} \\
&+\frac{\partial^2 TC}{\partial X_L^2}X_L^2 - 2\frac{\partial^2 TC}{\partial X_L^2}X_L X_L^0 + \frac{\partial^2 TC}{\partial X_L^2}X_L^{0\,2} + 2\frac{\partial^2 TC}{\partial X_L \partial z_v}\left(z_v - z_v^0\right)X_L \\
&-2\frac{\partial^2 TC}{\partial X_L \partial z_v}\left(z_v - z_v^0\right)X_L^0 + \left(z_v - z_v^0\right)'\frac{\partial^2 TC}{\partial z_v^2}\left(z_v - z_v^0\right)
\end{aligned}
$$

All terms that do not vary with X_L are considered as fixed transaction costs:

$$
\begin{aligned}
(27)\quad TC_f^* &= f\begin{pmatrix} X_L^0 \\ z_v^0 \end{pmatrix} - \frac{\partial TC}{\partial X_L}X_L^0 + \left(z_v - z_v^0\right)'\frac{\partial TC}{\partial z_v} + \frac{\partial^2 TC}{\partial X_L^2}X_L^{0\,2} \\
&-2\frac{\partial^2 TC}{\partial X_L \partial z_v}\left(z_v - z_v^0\right)X_L^0 + \left(z_v - z_v^0\right)'\frac{\partial^2 TC}{\partial z_v^2}\left(z_v - z_v^0\right)
\end{aligned}
$$

[2]All derivatives are evaluated at point $\begin{pmatrix} X_L^0 \\ z_v^0 \end{pmatrix}$ but in the following this is omitted for better readability.

Now we get for the variable transaction costs

$$\text{(28)} \quad TC_v^* = TC^* - TC_f^*$$

$$\text{(29)} \quad = \frac{\partial TC}{\partial X_L} X_L + \frac{\partial^2 TC}{\partial X_L^2} X_L^2 - 2\frac{\partial^2 TC}{\partial X_L^2} X_L X_L^0 + 2\frac{\partial^2 TC}{\partial X_L \partial \boldsymbol{z}_v} \left(\boldsymbol{z}_v - \boldsymbol{z}_v^0\right) X_L$$

$$\text{(30)} \quad = \left(\frac{\partial TC}{\partial X_L} - 2\frac{\partial^2 TC}{\partial X_L^2} X_L^0 - 2\frac{\partial^2 TC}{\partial X_L \partial \boldsymbol{z}_v} \boldsymbol{z}_v^0 + 2\frac{\partial^2 TC}{\partial X_L \partial \boldsymbol{z}_v} \boldsymbol{z}_v\right) X_L + \frac{\partial^2 TC}{\partial X_L^2} X_L^2$$

$$\text{(31)} \quad = \left(\widetilde{\boldsymbol{z}}_v' \boldsymbol{\beta}_v\right) X_L + \beta_{v1} X_L^2$$

with

$$\text{(32)} \quad \widetilde{\boldsymbol{z}}_v = \begin{pmatrix} 1 \\ \boldsymbol{z}_v \end{pmatrix}$$

$$\text{(33)} \quad \boldsymbol{\beta}_v = \begin{pmatrix} \frac{\partial TC}{\partial X_L} - 2\frac{\partial^2 TC}{\partial X_L^2} X_L^0 - 2\frac{\partial^2 TC}{\partial X_L \partial \boldsymbol{z}_v} \boldsymbol{z}_v^0 \\ 2\frac{\partial^2 TC}{\partial X_L \partial \boldsymbol{z}_v} \end{pmatrix}$$

$$\text{(34)} \quad \beta_{v1} = \frac{\partial^2 TC}{\partial X_L^2}$$

4.4 Exclusion Variables

In a two-step Heckman estimation, the variables that are regressors in the first-step selection equation (say, $\boldsymbol{x}_1$) but are not regressors in the second-step regression equation (say, $\boldsymbol{x}_2$) are called "exclusion variables." If there are no exclusion variables ($\boldsymbol{x}_1 \subseteq \boldsymbol{x}_2$), the sample correction term in the second step (say, λ) is likely to be highly correlated with the other regressors in $\boldsymbol{x}_2$ because λ is a (non-linear) function of a linear combination of the variables in $\boldsymbol{x}_1$ ($\lambda = \phi(\boldsymbol{x}_1{}'\boldsymbol{\gamma})/\Phi(\boldsymbol{x}_1{}'\boldsymbol{\gamma})$, where $\boldsymbol{\gamma}$ are the coefficients of the selection equation and ϕ and Φ are probability density function (pdf) and the cumulative distribution function (cdf) of the standard normal distribution, respectively). Hence, the purpose of exclusion variables is to reduce the correlation among the regressors (multicollinearity) in the second-step estimation. Although high multicollinearity does not result in biased estimates, it leads to large standard errors, which means that the estimates are rather imprecise.

The exclusion variables for the equations explaining the shadow price of labor can be identified from table 4 in the main article. The exclusion variables for the marginal revenue of labor supply (equation (24) in the main article) are the number of kids (N_k), land and capital endowment of the farm (R_g, R_k); the capital intensity on the farm (R_k/R_g); and the prices of farm netputs (P_c, P_a, P_v). The exclusion variables for the marginal cost of labor demand (equation (25) in the main article) are the age pattern of the household (N_k, N_w, N_o); sex, age, and age squared of the head of the household (D_f, A_h, A_h^2); land and capital endowment of the farm (R_g, R_k); and the prices of farm netputs (P_c, P_a, P_v).

The exclusion variables for the equations explaining the quantity of supplied labor (equations (26) and (27) in the main article) are variables that are in $\boldsymbol{z}$ but not in $\boldsymbol{z}_x^b$ and $\boldsymbol{z}_x^s$, respectively. The exclusion variables for the equations explaining the quantity of hired labor (equations (28) and (29) in the main article) are variables that are in $\boldsymbol{z}$ but not in $\boldsymbol{z}_x^b$ and $\boldsymbol{z}_x^h$, respectively. Theoretically, the exclusion variables in (26) and (28) are the variables that are in $\boldsymbol{z}_f^s$ or $\boldsymbol{z}_f^h$ but not in $\boldsymbol{z}^\pi$, $\boldsymbol{z}^u$, $\boldsymbol{z}^s$, or $\boldsymbol{z}^h$, the exclusion variables in (27) are the variables that are in $\boldsymbol{z}_f^s$, $\boldsymbol{z}_f^h$ or $\boldsymbol{z}^h$ but not in $\boldsymbol{z}^\pi$, $\boldsymbol{z}^u$ or $\boldsymbol{z}^s$, and the exclusion variables in (29) are the variables that are in $\boldsymbol{z}_f^s$, $\boldsymbol{z}_f^h$ or $\boldsymbol{z}^s$ but not in $\boldsymbol{z}^\pi$, $\boldsymbol{z}^u$ or $\boldsymbol{z}^h$. However, in practice, our data set does not include any variables that influence fixed transaction costs ($\boldsymbol{z}_f^s$, $\boldsymbol{z}_f^h$) but do not influence variable transaction costs or the average skill level ($\boldsymbol{z}^s$, $\boldsymbol{z}^h$). Thus, given the specification of the $\boldsymbol{z}$ variables in section "Data and Empirical Results" in the main article, we have an exclusion variable only in (27) (R_K/R_g) but not in the other three X equations. Although this leads to multicollinearity, it does not matter in our special case because we are interested in the fitted values but not the estimated coefficients. As long as multicollinearity is not so high that it rules out estimation, we can calculate fitted values that are orthogonal to the error terms of the estimations of the shadow price of labor (given that the regressors are not correlated with these error terms, too).

5 Assumptions about Error Terms

We assume that the residuals of the participation equations (22, 23) in the main article, ε^s and ε^h, have a bivariate normal distribution:

$$(35) \quad \begin{pmatrix} \varepsilon^s \\ \varepsilon^h \end{pmatrix} \sim N\left(\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{bmatrix} 1 & \rho \\ & 1 \end{bmatrix} \right)$$

Further, we assume a joint normal distribution of ε^s, ε^h, $\widetilde{v}^s$ and $\widetilde{v}^h$ with covariances $\sigma^s = cov(\widetilde{v}^s, \varepsilon^s)$ and $\sigma^h = cov(\widetilde{v}^h, \varepsilon^h)$, where $\widetilde{v}^s$ and $\widetilde{v}^h$ would be the error terms of equations (24) and (25) in the main article, respectively, without selectivity terms. From this we can obtain the conditional expectation of the error terms

$$(36) \qquad E\left[\widetilde{v}^s | Y^{s*} > 0\right] = \sigma^s \lambda^s$$

$$(37) \qquad E\left[\widetilde{v}^h | Y^{h*} > 0\right] = \sigma^h \lambda^h$$

where λ^s and λ^h are defined as in equation (30) of the main article.

Furthermore, we assume a joint normal distribution of ε^s, ε^h, $\widetilde{\xi}_s^b$, $\widetilde{\xi}_s^s$, $\widetilde{\xi}_h^b$, and $\widetilde{\xi}_h^h$ with covariances

$\sigma_s^{bs} = cov(\widetilde{\xi}_s^b, \varepsilon^s)$, $\sigma_s^{bh} = cov(\widetilde{\xi}_s^b, \varepsilon^h)$, $\sigma_s^{ss} = cov(\widetilde{\xi}_s^s, \varepsilon^s)$, $\sigma_s^{sh} = cov(\widetilde{\xi}_s^s, \varepsilon^h)$, $\sigma_h^{bs} = cov(\widetilde{\xi}_h^b, \varepsilon^s)$, $\sigma_h^{bh} = cov(\widetilde{\xi}_h^b, \varepsilon^h)$, $\sigma_h^{hs} = cov(\widetilde{\xi}_h^h, \varepsilon^s)$, and $\sigma_h^{hh} = cov(\widetilde{\xi}_h^h, \varepsilon^h)$, where $\widetilde{\xi}_s^b$, $\widetilde{\xi}_s^s$, $\widetilde{\xi}_h^b$, and $\widetilde{\xi}_h^h$ would be the error terms of equations (26), (27), (28), and (29) in the main article, respectively, without selectivity terms. From this we can obtain the conditional expectation of the error terms

$$(38)\qquad E\left[\widetilde{\xi}_s^b | Y^{s*} > 0 \wedge Y^{h*} > 0\right] = \sigma_s^{bs}\lambda^{bs} + \sigma_s^{bh}\lambda^{bh}$$

$$(39)\qquad E\left[\widetilde{\xi}_s^s | Y^{s*} > 0 \wedge Y^{h*} \leq 0\right] = \sigma_s^{ss}\lambda^{ss} + \sigma_s^{sh}\lambda^{sh}$$

$$(40)\qquad E\left[\widetilde{\xi}_h^b | Y^{s*} > 0 \wedge Y^{h*} > 0\right] = \sigma_h^{bs}\lambda^{bs} + \sigma_h^{bh}\lambda^{bh}$$

$$(41)\qquad E\left[\widetilde{\xi}_h^h | Y^{s*} \leq 0 \wedge Y^{h*} > 0\right] = \sigma_h^{hs}\lambda^{hs} + \sigma_h^{hh}\lambda^{hh}$$

where the λs are defined as in equations (31) to (33) of the main article.

6 Proof of Selectivity Terms

In the following we derive the selectivity terms used in our 2SLS/IV estimation procedure.

To this end we consider a trivariate normal distribution of the variables X_1, X_2 and X_3 with density function $\phi_3(X_1, X_2, X_3)$, mean vector $\boldsymbol{\mu}$ and covariance matrix $\boldsymbol{\Sigma}$, where it holds:

$$(42)\quad \boldsymbol{\mu} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}; \qquad \boldsymbol{\Sigma} = \begin{pmatrix} \sigma_1^2 & \sigma_{12} & \sigma_{13} \\ & 1 & \rho \\ & & 1 \end{pmatrix}$$

The corresponding marginal normal distributions of the variables X_2 and X_3 are bivariate normal distributed with density function $\phi_2(X_1, X_2)$, mean vector $\boldsymbol{\mu}_{23}$ and covariance matrix $\boldsymbol{\Sigma}_{23}$, where it holds (see for example Greene 2003):

$$(43)\quad \boldsymbol{\mu}_{23} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}; \qquad \boldsymbol{\Sigma}_{23} = \begin{pmatrix} 1 & \rho \\ & 1 \end{pmatrix}$$

The corresponding conditional distribution of X_1 has density function $\phi(X_1 | X_2, X_3)$, mean μ_1^*, and variance σ_1^{2*}, where it holds (see for example Greene 2003):

$$(44)\quad \mu_1^* = \frac{(\sigma_{12} - \rho\sigma_{13})X_2 + (\sigma_{13} - \rho\sigma_{12})X_3}{1 - \rho^2}$$

$$(45)\quad \sigma_1^{2*} = \sigma_1^2 - \frac{\sigma_{12}^2 - 2\sigma_{12}\sigma_{13}\rho + \sigma_{13}^2}{1 - \rho^2}$$

Given the definitions above we first prove the following three Lemmas

Lemma 1:

For $a_2, a_3 \in \mathbb{R}$ it holds

$$\int_{a_3}^{\infty} \phi(X_3)\, \phi\left(\frac{a_2 - \rho X_3}{\sqrt{1-\rho^2}}\right) \mathrm{d}X_3 = \sqrt{1-\rho^2}\phi(a_2)\, \Phi\left(\frac{-a_3 + \rho a_2}{\sqrt{1-\rho^2}}\right) \tag{46}$$

Proof:

$$\begin{aligned}
& \int_{a_3}^{\infty} \phi(X_3)\, \phi\left(\frac{a_2 - \rho X_3}{\sqrt{1-\rho^2}}\right) \mathrm{d}X_3 \\
(47) \quad = & \int_{a_3}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}X_3^2} \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{a_2-\rho X_3}{\sqrt{1-\rho^2}}\right)^2} \mathrm{d}X_3 \\
(48) \quad = & \int_{a_3}^{\infty} \frac{1}{2\pi} e^{-\frac{1}{2}\left(X_3^2 + \frac{(a_2-\rho X_3)^2}{1-\rho^2}\right)} \mathrm{d}X_3 \\
(49) \quad = & \int_{a_3}^{\infty} \frac{1}{2\pi} e^{-\frac{1}{2}\left(\frac{X_3^2\left(1-\rho^2\right)}{1-\rho^2} + \frac{a_2^2 - 2a_2\rho X_3 + \rho^2 X_3^2}{1-\rho^2}\right)} \mathrm{d}X_3 \\
(50) \quad = & \int_{a_3}^{\infty} \frac{1}{2\pi} e^{-\frac{1}{2}\left(\frac{a_2^2\left(1-\rho^2\right)}{1-\rho^2} + \frac{x_3^2 - 2x_3\rho a_2 + \rho^2 a_2^2}{1-\rho^2}\right)} \mathrm{d}X_3 \\
(51) \quad = & \int_{a_3}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}a_2^2} \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x_3-\rho a_2}{\sqrt{1-\rho^2}}\right)^2} \mathrm{d}X_3 \\
(52) \quad = & \int_{a_3}^{\infty} \phi(a_2)\, \phi\left(\frac{x_3 - \rho a_2}{\sqrt{1-\rho^2}}\right) \mathrm{d}X_3 \\
(53) \quad = & \phi(a_2)\sqrt{1-\rho^2} \int_{a_3}^{\infty} \frac{1}{\sqrt{1-\rho^2}} \phi\left(\frac{x_3 - \rho a_2}{\sqrt{1-\rho^2}}\right) \mathrm{d}X_3 \\
(54) \quad = & \phi(a_2)\sqrt{1-\rho^2} \int_{\frac{a_3-\rho a_2}{\sqrt{1-\rho^2}}}^{\infty} \phi(Z_3)\, \mathrm{d}Z_3 \\
(55) \quad = & \sqrt{1-\rho^2}\phi(a_2)\, \Phi\left(\frac{-a_3 + \rho a_2}{\sqrt{1-\rho^2}}\right)
\end{aligned}$$

q.e.d.

Corollary to Lemma 1:

$$\int_{-\infty}^{a_3} \phi(X_3)\, \phi\left(\frac{a_2 - \rho X_3}{\sqrt{1-\rho^2}}\right) \mathrm{d}X_3 = \sqrt{1-\rho^2}\phi(a_2)\, \Phi\left(\frac{a_3 - \rho a_2}{\sqrt{1-\rho^2}}\right) \tag{56}$$

Lemma 2

For $a_2, a_3 \in \mathbb{R}$ it holds

$$\begin{aligned} & \int_{a_3}^{\infty} X_3\,\phi\left(X_3\right)\Phi\left(\frac{-a_2+\rho X_3}{\sqrt{1-\rho^2}}\right)\mathrm{d}X_3 \\ (57) \quad = \; & \phi\left(a_3\right)\Phi\left(\frac{-a_2+\rho a_3}{\sqrt{1-\rho^2}}\right)+\rho\,\phi\left(a_2\right)\Phi\left(\frac{-a_3+\rho a_2}{\sqrt{1-\rho^2}}\right) \end{aligned}$$

Proof:

$$\begin{aligned} & \int_{a_3}^{\infty} X_3\,\phi\left(X_3\right)\Phi\left(\frac{-a_2+\rho X_3}{\sqrt{1-\rho^2}}\right)\mathrm{d}X_3 \\ (58) \quad = \; & \int_{a_3}^{\infty} g'\left(X_3\right) f\left(X_3\right)\mathrm{d}X_3 \end{aligned}$$

with

$$\begin{aligned} (59) \quad g'\left(X_3\right) &= X_3\,\phi\left(X_3\right) \\ (60) \quad f\left(X_3\right) &= \Phi\left(\frac{-a_2+\rho X_3}{\sqrt{1-\rho^2}}\right) \end{aligned}$$

From partial integration it follows

$$\begin{aligned} & \int_{a_3}^{\infty} g'\left(X_3\right) f\left(X_3\right)\mathrm{d}X_3 \\ (61) \quad = \; & \lim_{a\to\infty} g\left(a\right) f\left(a\right)-g\left(a_3\right) f\left(a_3\right)-\int_{a_3}^{\infty} g\left(X_3\right) f'\left(X_3\right)\mathrm{d}X_3 \end{aligned}$$

with

$$\begin{aligned} (62) \quad g\left(X_3\right) &= -\phi\left(X_3\right) \\ (63) \quad f'\left(X_3\right) &= \phi\left(\frac{-a_2+\rho X_3}{\sqrt{1-\rho^2}}\right) \end{aligned}$$

substituting (62) and (63) into (61) we get

$$\lim_{a\to\infty} g(a)\, f(a) - g(a_3)\, f(a_3) - \int_{a_3}^{\infty} g(X_3)\, f'(X_3)\, \mathrm{d}X_3$$

$$= \phi(a_3)\,\Phi\left(\frac{-a_2+\rho a_3}{\sqrt{1-\rho^2}}\right) + \frac{\rho}{\sqrt{1-\rho^2}} \int_{a_3}^{\infty} \phi(X_3)\, \phi\left(\frac{-a_2+\rho X_3}{\sqrt{1-\rho^2}}\right) \mathrm{d}X_3 \tag{64}$$

$$= \phi(a_3)\,\Phi\left(\frac{-a_2+\rho a_3}{\sqrt{1-\rho^2}}\right) + \frac{\rho}{\sqrt{1-\rho^2}} \int_{a_3}^{\infty} \phi(X_3)\, \phi\left(\frac{a_2-\rho X_3}{\sqrt{1-\rho^2}}\right) \mathrm{d}X_3 \tag{65}$$

applying Lemma 1 results in

$$\phi(a_3)\,\Phi\left(\frac{-a_2+\rho a_3}{\sqrt{1-\rho^2}}\right) + \frac{\rho}{\sqrt{1-\rho^2}} \int_{a_3}^{\infty} \phi(X_3)\, \phi\left(\frac{a_2-\rho X_3}{\sqrt{1-\rho^2}}\right) \mathrm{d}X_3$$

$$= \phi(a_3)\,\Phi\left(\frac{-a_2+\rho a_3}{\sqrt{1-\rho^2}}\right) + \rho\, \phi(a_2)\,\Phi\left(\frac{-a_3+\rho a_2}{\sqrt{1-\rho^2}}\right) \tag{66}$$

q.e.d.

Corollary to Lemma 2:

$$\int_{-\infty}^{a_3} X_3\, \phi(X_3)\, \Phi\left(\frac{-a_2+\rho X_3}{\sqrt{1-\rho^2}}\right) \mathrm{d}X_3$$

$$= -\phi(a_3)\, \Phi\left(\frac{-a_2+\rho a_3}{\sqrt{1-\rho^2}}\right) + \rho\, \phi(a_2)\, \Phi\left(\frac{a_3-\rho a_2}{\sqrt{1-\rho^2}}\right) \tag{67}$$

Lemma 3

For $a_2, a_3 \in \mathbb{R}$ it holds:

$$\int_{-\infty}^{\infty} \int_{a_2}^{\infty} \int_{a_3}^{\infty} \phi_3(X_1, X_2, X_3)\, \mathrm{d}X_3\, \mathrm{d}X_2\, \mathrm{d}X_1 = \Phi_2(-a_2, -a_3, \boldsymbol{\Sigma}_{23}) \tag{68}$$

Corollary to Lemma 3:

$$\int_{-\infty}^{\infty} \int_{a_2}^{\infty} \int_{-\infty}^{a_3} \phi_3(X_1, X_2, X_3)\, \mathrm{d}X_3\, \mathrm{d}X_2\, \mathrm{d}X_1 = \Phi_2\left(-a_2, a_3, \left(1-\rho^2\right) \boldsymbol{\Sigma}_{23}^{-1}\right) \tag{69}$$

Lemma 4:

$$\int X_2 \phi(X_2)\, \mathrm{d}X_2 = -\phi(X_2) \tag{70}$$

Proof:

$$(71)\quad \frac{\partial \phi(X_2)}{\partial X_2} = -X_2\,\phi(X_2)$$

q.e.d.

Theorem

Given a trivariate normal distribution as defined above. Then it holds for any $a_2, a_3 \in \mathbb{R}$:

(i) $E(X_1 | X_2 > a_2 \wedge X_3 > a_3)$

$$(72)\quad = \frac{\sigma_{13}\phi(a_3)\,\Phi\left(\frac{-a_2+\rho a_3}{\sqrt{1-\rho^2}}\right) + \sigma_{12}\phi(a_2)\,\Phi\left(\frac{-a_3+\rho a_2}{\sqrt{1-\rho^2}}\right)}{\Phi_2(-a_2,-a_3,\boldsymbol{\Sigma}_{23})}$$

(ii) $E(X_1 | X_2 > a_2 \wedge X_3 < a_3)$

$$(73)\quad = \frac{-\sigma_{13}\phi(a_3)\,\Phi\left(\frac{-a_2+\rho a_3}{\sqrt{1-\rho^2}}\right) + \sigma_{12}\phi(a_2)\,\Phi\left(\frac{a_3-\rho a_2}{\sqrt{1-\rho^2}}\right)}{\Phi_2\left(-a_2,a_3,(1-\rho^2)\boldsymbol{\Sigma}_{23}^{-1}\right)}$$

Proof of (i):

It holds the definition

$$(74)\quad E(X_1 | X_2 > a_2 \wedge X_3 > a_3) = \frac{\int_{-\infty}^{\infty}\int_{a_2}^{\infty}\int_{a_3}^{\infty} X_1\,\phi_3(X_1,X_2,X_3)\,\mathrm{d}X_3\,\mathrm{d}X_2\,\mathrm{d}X_1}{\int_{-\infty}^{\infty}\int_{a_2}^{\infty}\int_{a_3}^{\infty} \phi_3(X_1,X_2,X_3)\,\mathrm{d}X_3\,\mathrm{d}X_2\,\mathrm{d}X_1}$$

Applying Lemma 3 results in

$$(75)\quad E(X_1 | X_2 > a_2 \wedge X_3 > a_3) = \frac{\int_{-\infty}^{\infty}\int_{a_2}^{\infty}\int_{a_3}^{\infty} X_1\,\phi_3(X_1,X_2,X_3)\,\mathrm{d}X_3\,\mathrm{d}X_2\,\mathrm{d}X_1}{\Phi_2(-a_2,-a_3,\boldsymbol{\Sigma}_{23})}$$

Now it holds for any trivariate normal distribution

$$\int_{-\infty}^{\infty}\int_{a_2}^{\infty}\int_{a_3}^{\infty} X_1\,\phi_3(X_1,X_2,X_3)\,\mathrm{d}X_3\,\mathrm{d}X_2\,\mathrm{d}X_1$$

$$(76)\quad = \int_{a_2}^{\infty}\int_{a_3}^{\infty} \phi_2(X_2,X_3)\int_{-\infty}^{\infty} X_1\,\phi_3(X_1 | X_2,X_3)\,\mathrm{d}X_1\,\mathrm{d}X_3\,\mathrm{d}X_2$$

$$(77)\quad = \int_{a_2}^{\infty}\int_{a_3}^{\infty} \phi_2(X_2,X_3)\,\mu_1^*\,\mathrm{d}X_3\,\mathrm{d}X_2$$

$$(78)\quad = \int_{a_2}^{\infty}\int_{a_3}^{\infty} \phi_2(X_2,X_3)\,\frac{(\sigma_{12}-\rho\sigma_{13})X_2 + (\sigma_{13}-\sigma_{12}\rho)X_3}{1-\rho^2}\,\mathrm{d}X_3\,\mathrm{d}X_2$$

$$(79)\quad = \int_{a_2}^{\infty}\int_{a_3}^{\infty} \phi(X_3)\,\phi(X_2 | X_3)\,(K_2X_2 + K_3X_3)\,\mathrm{d}X_3\,\mathrm{d}X_2$$

with

$$K_2 = \frac{\sigma_{12} - \rho\sigma_{13}}{1-\rho^2} \tag{80}$$

$$K_3 = \frac{\sigma_{13} - \rho\sigma_{12}}{1-\rho^2} \tag{81}$$

Now it holds

$$\int_{a_2}^{\infty}\int_{a_3}^{\infty} \phi(X_3) \frac{1}{\sqrt{1-\rho^2}} \phi\left(\frac{X_2-\rho X_3}{\sqrt{1-\rho^2}}\right)(K_2X_2+K_3X_3)\,\mathrm{d}X_3\,\mathrm{d}X_2$$

$$= \int_{a_3}^{\infty} \phi(X_3)\left(K_2\int_{a_2}^{\infty} X_2 \frac{1}{\sqrt{1-\rho^2}} \phi\left(\frac{X_2-\rho X_3}{\sqrt{1-\rho^2}}\right)\mathrm{d}X_2 \right.$$
$$\left. + K_3X_3\int_{a_2}^{\infty}\frac{1}{\sqrt{1-\rho^2}} \phi\left(\frac{X_2-\rho X_3}{\sqrt{1-\rho^2}}\right)\mathrm{d}X_2\right)\mathrm{d}X_3 \tag{82}$$

$$= \int_{a_3}^{\infty} \phi(X_3)\left(K_2\sqrt{1-\rho^2}\int_{a_2}^{\infty} \frac{1}{\sqrt{1-\rho^2}}\frac{X_2-\rho X_3}{\sqrt{1-\rho^2}} \phi\left(\frac{X_2-\rho X_3}{\sqrt{1-\rho^2}}\right)\mathrm{d}X_2 \right.$$
$$\left. + (K_2\rho+K_3)X_3\int_{a_2}^{\infty}\frac{1}{\sqrt{1-\rho^2}} \phi\left(\frac{X_2-\rho X_3}{\sqrt{1-\rho^2}}\right)\mathrm{d}X_2\right)\mathrm{d}X_3 \tag{83}$$

$$= \int_{a_3}^{\infty} \phi(X_3)\left(K_2\sqrt{1-\rho^2}\int_{\frac{a_2-\rho X_3}{\sqrt{1-\rho^2}}}^{\infty} Z_2\phi(Z_2)\,\mathrm{d}Z_2 \right.$$
$$\left. + (K_2\rho+K_3)X_3\int_{\frac{a_2-\rho X_3}{\sqrt{1-\rho^2}}}^{\infty}\phi(Z_2)\,\mathrm{d}Z_2\right)\mathrm{d}X_3 \tag{84}$$

applying Lemma 4 we get

$$\int_{a_3}^{\infty} \phi(X_3) K_2\sqrt{1-\rho^2}\phi\left(\frac{a_2-\rho X_3}{\sqrt{1-\rho^2}}\right)\mathrm{d}X_3$$
$$+\int_{a_3}^{\infty} \phi(X_3)\,(K_2\rho+K_3)\,X_3\Phi\left(\frac{-a_2+\rho X_3}{\sqrt{1-\rho^2}}\right)$$
$$= K_2\sqrt{1-\rho^2}\int_{a_3}^{\infty}\phi(X_3)\,\phi\left(\frac{a_2-\rho X_3}{\sqrt{1-\rho^2}}\right)\mathrm{d}X_3 \tag{85}$$
$$+(K_2\rho+K_3)\int_{a_3}^{\infty} X_3\,\phi(X_3)\,\Phi\left(\frac{-a_2+\rho X_3}{\sqrt{1-\rho^2}}\right)\mathrm{d}X_3$$

applying Lemma 1 and Lemma 2 we get

$$
\begin{aligned}
& K_2\sqrt{1-\rho^2}\sqrt{1-\rho^2}\,\phi(a_2)\,\Phi\left(\frac{-a_3+\rho a_2}{\sqrt{1-\rho^2}}\right) \\
& +(K_2\rho+K_3)\left(\phi(a_3)\,\Phi\left(\frac{-a_2+\rho a_3}{\sqrt{1-\rho^2}}\right)+\rho\,\phi(a_2)\,\Phi\left(\frac{-a_3+\rho a_2}{\sqrt{1-\rho^2}}\right)\right) \\
(86)\qquad = \; & \left(K_2\left(1-\rho^2\right)+(K_2\rho+K_3)\rho\right)\phi(a_2)\Phi\left(\frac{-a_3+\rho a_2}{\sqrt{1-\rho^2}}\right) \\
& +(K_2\rho+K_3)\,\phi(a_3)\,\Phi\left(\frac{-a_2+\rho a_3}{\sqrt{1-\rho^2}}\right) \\
(87)\qquad = \; & (K_2+K_3\rho)\,\phi(a_2)\Phi\left(\frac{-a_3+\rho a_2}{\sqrt{1-\rho^2}}\right) \\
& +(K_2\rho+K_3)\,\phi(a_3)\,\Phi\left(\frac{-a_2+\rho a_3}{\sqrt{1-\rho^2}}\right)
\end{aligned}
$$

substituting (81) and (80) for K_2 and K_3

$$
\begin{aligned}
& \left(\frac{\sigma_{12}-\rho\sigma_{13}}{1-\rho^2}+\frac{\sigma_{13}-\rho\sigma_{12}}{1-\rho^2}\rho\right)\phi(a_2)\Phi\left(\frac{-a_3+\rho a_2}{\sqrt{1-\rho^2}}\right) \\
& +\left(\frac{\sigma_{12}-\rho\sigma_{13}}{1-\rho^2}\rho+\frac{\sigma_{13}-\rho\sigma_{12}}{1-\rho^2}\right)\phi(a_3)\,\Phi\left(\frac{-a_2+\rho a_3}{\sqrt{1-\rho^2}}\right) \\
(88)\qquad = \; & \left(\frac{\sigma_{12}-\rho\sigma_{13}+\sigma_{13}\rho-\rho^2\sigma_{12}}{1-\rho^2}\right)\phi(a_2)\Phi\left(\frac{-a_3+\rho a_2}{\sqrt{1-\rho^2}}\right) \\
& +\left(\frac{\rho\sigma_{12}-\rho^2\sigma_{13}+\sigma_{13}-\rho\sigma_{12}}{1-\rho^2}\right)\phi(a_3)\,\Phi\left(\frac{-a_2+\rho a_3}{\sqrt{1-\rho^2}}\right) \\
(89)\qquad = \; & \sigma_{12}\phi(a_2)\Phi\left(\frac{-a_3+\rho a_2}{\sqrt{1-\rho^2}}\right)+\sigma_{13}\phi(a_3)\,\Phi\left(\frac{-a_2+\rho a_3}{\sqrt{1-\rho^2}}\right)
\end{aligned}
$$

q.e.d.

Proof of (ii):
This proof is analogous to the proof of (i) except that the Corollaries are applied in place of the Lemmas.

7 Formulas to Calculate Farm-Household Elasticities

7.1 Notations

Price Elasticities on Production Side

$\varepsilon_{ij} = \frac{\partial X_i}{\partial P_j}\frac{P_j}{X_i}$ = traditional price elasticity of netput i with respect to price of netput j

$\varepsilon_{ij}^{FHM} = \frac{\partial X_i}{\partial P_j}\frac{P_j}{X_i}$ = FHM price elasticity of netput i with respect to price of netput/good j

Price Elasticities on Consumption Side

$\Theta_{ij} = \frac{\partial C_i}{\partial P_j}\frac{P_j}{C_i}$ = traditional Marshallian price elasticity of good i with respect to price of good j

$\Theta_{ij}^{H} = \frac{\partial C_i^H}{\partial P_j}\frac{P_j}{C_i}$ = traditional Hicksian price elasticity of good i with respect to price of good j

$\eta_i = \frac{\partial C_i}{\partial Y}\frac{Y}{C_i}$ = traditional income elasticity of good i

$\Theta_{ij}^{FHM} = \frac{\partial C_i}{\partial P_j}\frac{P_j}{C_i}$ = FHM price elasticity of good i with respect to price of netput/good j

Price Elasticities of Labor Allocation

$\varphi_{sL} = \frac{\partial X_L^s}{\partial P_L^s}\frac{P_L^s}{X_L^s}$ = traditional price elasticity of supplied labor with respect to labor price

$\varphi_{hL} = \frac{\partial X_L^h}{\partial P_L^h}\frac{P_L^h}{X_L^h}$ = traditional price elasticity of hired labor with respect to labor price

$\varphi_{sj}^{FHM} = \frac{\partial X_L^s}{\partial P_j}\frac{P_j}{X_L^s}$ = FHM price elasticity of supplied labor with respect to price of netput/good j

$\varphi_{hj}^{FHM} = \frac{\partial X_L^h}{\partial P_j}\frac{P_j}{X_L^h}$ = FHM price elasticity of hired labor with respect to price of netput/good j

$\varphi_{nj}^{FHM} = \frac{\partial X_L^n}{\partial P_j}\frac{P_j}{X_L^n}$ = FHM price elasticity of net supplied labor with respect to price of netput/good j

$\varphi_{fj}^{FHM} = \frac{\partial X_L^f}{\partial P_j}\frac{P_j}{X_L^f}$ = FHM price elasticity of family labor on the farm with respect to price of netput/good j

Shadow Price Elasticity of Labor

$\Psi_j = \frac{\partial P_L^*}{\partial P_j}\frac{P_j}{P_L^*}$ = elasticity of the shadow price of labor with respect to price of netput/good j

7.2 Price Elasticities of the Separable Household Models

Price Elasticities on Production Side

The price elasticities on production side are simply the traditional price elasticities:

$$(90)\quad \varepsilon_{ij}^{FHM} = \varepsilon_{ij} \quad \forall i,j \in \{a,c,v,L\}$$
$$(91)\quad \varepsilon_{im}^{FHM} = 0 \quad \forall i \in \{a,c,v,L\}$$

Price Elasticities on Consumption Side

The price elasticities on consumption side consist of the normal Marshallian price effect and of an income effect due to an income change from farming or from working off-farm:

$$(92)\quad \Theta_{ij}^{sFHM} = \left.\frac{\partial C_i}{\partial P_j}\right|_{Y=\text{const.}}\frac{P_j}{C_i} + \frac{\partial C_i}{\partial Y}\frac{\partial Y}{\partial P_j}\frac{P_j}{C_i}$$
$$(93)\quad = \frac{\partial C_i^H}{\partial P_j}\frac{P_j}{C_i} + \frac{\partial C_i}{\partial Y}\frac{Y}{C_i}\left(\frac{\partial Y}{\partial P_j} - C_j\right)\frac{P_j}{Y}$$
$$(94)\quad = \Theta_{ij}^H + \eta_i\left(\frac{\partial Y}{\partial P_j} - C_j\right)\frac{P_j}{Y}$$

Evaluating $\frac{\partial Y}{\partial P_j}$ and removing all terms that are zero, we get the elasticities for each of the prices:

$$(95)\quad \Theta_{ij}^{sFHM} = \eta_i\frac{P_jX_j}{Y} \quad \forall i \in \{m,a,L\},\ j \in \{c,v\}$$
$$(96)\quad \Theta_{ia}^{sFHM} = \Theta_{ia}^H + \eta_i\frac{P_a(X_a - C_a)}{Y} \quad \forall i \in \{m,a,L\}$$
$$(97)\quad \Theta_{iL}^{sFHM} = \Theta_{iL}^H + \eta_i\frac{P_j\left(X_L^s - X_L^h\right)}{Y} \quad \forall i \in \{m,a,L\}$$
$$(98)\quad \Theta_{im}^{sFHM} = \Theta_{im}^H - \eta_i\frac{P_mC_m}{Y} \quad \forall i \in \{m,a,L\}$$

Price Elasticity of Net Supply of Labor

The price elasticity of net supply of labor is calculated residually:

$$
\begin{aligned}
(99)\quad \varphi_{nj}^{sFHM} &= \frac{\partial\left(X_L^s - X_L^h\right)}{\partial P_j}\frac{P_j}{X_L^{sn}} \\
(100)\quad &= \frac{\partial\left(T_L + X_L - C_L\right)}{\partial P_j}\frac{P_j}{X_L^n} \\
(101)\quad &= \frac{\partial X_L}{\partial P_j}\frac{P_j}{X_L}\frac{X_L}{X_L^n} - \frac{\partial C_L}{\partial P_j}\frac{P_j}{C_L}\frac{C_L}{X_L^n} \\
(102)\quad &= \varepsilon_{Lj}^{FHM}\frac{X_L}{X_L^n} - \Theta_{Lj}^{FHM}\frac{C_L}{X_L^n} \quad \forall\, j \in \{a,c,v,L,m\}
\end{aligned}
$$

7.3 Price Elasticities of the Non-separable Household Models

The following formulas are valid for all four labor regimes. In case that the household does not supply labor, X_L^s and φ_L^s have to be set to zero and in case that the household does not hire labor, X_L^h and φ_L^h have to be set to zero.

Shadow Price Elasticities

We derive the shadow price elasticities from equation (14) of the main article:

$$
\begin{aligned}
(103)\quad \Psi_j &= \frac{-\frac{\partial X_L}{\partial P_j} + \left.\frac{\partial C_L}{\partial P_j}\right|_{Y=\text{const.}} + \frac{\partial C_L}{\partial Y}\frac{\partial Y}{\partial P_j}}{\frac{\partial X_L}{\partial P_L^*} + \frac{\partial X_L^h}{\partial P_L^*} - \frac{\partial X_L^s}{\partial P_L^*} - \frac{\partial C_L^H}{\partial P_L^*}}\frac{P_j}{P_L^*} \\
(104)\quad &= \frac{-\frac{\partial X_L}{\partial P_j} + \frac{\partial C_L^H}{\partial P_j} + \frac{\partial C_L}{\partial Y}\left(\frac{\partial Y}{\partial P_j} - C_j\right)}{\frac{\partial X_L}{\partial P_L^*} + \frac{\partial X_L^h}{\partial P_L^*} - \frac{\partial X_L^s}{\partial P_L^*} - \frac{\partial C_L^H}{\partial P_L^*}}\frac{P_j}{P_L^*} \\
(105)\quad &= \frac{-\frac{\partial X_L}{\partial P_j}\frac{P_j}{X_L}X_L + \frac{\partial C_L^H}{\partial P_j}\frac{P_j}{C_L}C_L + \frac{\partial C_L}{\partial Y}\frac{Y}{C_L}\left(\frac{\partial Y}{\partial P_j} - C_j\right)\frac{P_j}{Y}C_L}{\frac{\partial X_L}{\partial P_L^*}\frac{P_L^*}{X_L}X_L + \frac{\partial X_L^h}{\partial P_L^*}\frac{P_L^*}{X_L^h}X_L^h - \frac{\partial X_L^s}{\partial P_L^*}\frac{P_L^*}{X_L^s}X_L^s - \frac{\partial C_L^H}{\partial P_L^*}\frac{P_L^*}{C_L}C_L} \\
(106)\quad &= \frac{-\varepsilon_{Lj}X_L + \Theta_{Lj}^H C_L + \eta_L\left(\frac{\partial Y}{\partial P_j} - C_j\right)\frac{P_j}{Y}C_L}{\varepsilon_{LL}X_L + \varphi_L^h X_L^h - \varphi_L^s X_L^s - \Theta_{LL}^H C_L}
\end{aligned}
$$

Evaluating $\frac{\partial Y}{\partial P_j}$ and removing all terms that are zero, we get the elasticities for each of the exogenous prices:

$$(107)\ \Psi_j = \frac{-\varepsilon_{Lj}X_L + \eta_L \frac{P_j X_j}{Y} C_L}{\varepsilon_{LL}X_L + \varphi_L^h X_L^h - \varphi_L^s X_L^s - \Theta_{LL}^H C_L} \ \forall j \in \{c, v\}$$

$$(108)\ \Psi_a = \frac{-\varepsilon_{La}X_L + \Theta_{La}^H C_L + \eta_L \frac{P_a (X_a - C_a)}{Y} C_L}{\varepsilon_{LL}X_L + \varphi_L^h X_L^h - \varphi_L^s X_L^s - \Theta_{LL}^H C_L}$$

$$(109)\ \Psi_m = \frac{\Theta_{Lm}^H C_L - \eta_L \frac{P_L C_L}{Y} C_L}{\varepsilon_{LL}X_L + \varphi_L^h X_L^h - \varphi_L^s X_L^s - \Theta_{LL}^H C_L}$$

Given the convexity of the profit function $\Pi(.)$ in netput prices and the concavity of the expenditure function $e(.)$ in commodity prices and assuming that $g(.)$ is convex in X_L^h and $f(.)$ is concave in X_L^s, the denominator is always positive, because $\varphi_L^h = \left(\partial^2 g / \partial X_L^{h2}\right)^{-1} \left(P_L^h / X_L^h\right) \geq 0$, $X_L^h \geq 0$, $\varepsilon_{LL} = \left(\partial^2 \Pi / \partial {P_L}^2\right) (P_L / X_L) \leq 0$, $X_L \leq 0$, $\varphi_L^s = \left(\partial^2 f / \partial X_L^{s2}\right)^{-1} (P_L^s / X_L^s) \leq 0$, $X_L^s \geq 0$, $\Theta_{LL}^H = \left(\partial^2 e / \partial {P_L}^2\right) (C_L / P_L) \leq 0$, and $C_L \geq 0$.

Price Elasticities on Production Side

We derive the price elasticities on production side from equation (13) of the main article:

$$(110)\ \varepsilon_{ij}^{iFHM} = \left.\frac{\partial X_i}{\partial P_j}\right|_{P_L^* = \text{const.}} \frac{P_j}{X_i} + \frac{\partial X_i}{\partial P_L^*} \frac{P_L^*}{X_i} \frac{\partial P_L^*}{\partial P_j} \frac{P_j}{P_L}$$

$$(111)\ \qquad = \varepsilon_{ij}^{sFHM} + \varepsilon_{iL} \Psi_j$$

Substituting the direct component, which is the price elasticity of the separable model ε_{ij}^{sFHM}, we get the elasticities for each of the exogenous prices:

$$(112)\ \varepsilon_{ij}^{iFHM} = \varepsilon_{ij} + \varepsilon_{iL} \Psi_j \quad \forall i \in \{a, c, v, L\},\ j \in \{c, a, v\}$$

$$(113)\ \varepsilon_{im}^{iFHM} = \varepsilon_{iL} \Psi_m \quad \forall i \in \{a, c, v, L\}$$

Price Elasticities on Consumption Side

We derive the price elasticities on consumption side from equation (13) of the main article:

$$
\begin{aligned}
(114)\ \Theta_{ij}^{iFHM} &= \left.\frac{\partial C_i}{\partial P_j}\right|_{P_L^*=\text{const.}} \frac{P_j}{C_i} + \frac{\partial C_i}{\partial P_L^*}\frac{\partial P_L^*}{\partial P_j}\frac{P_j}{C_i} \\
(115)\qquad &= \left.\frac{\partial C_i}{\partial P_j}\right|_{P_L^*=\text{const.}} \frac{P_j}{C_i} + \left(\left.\frac{\partial C_i}{\partial P_L^*}\right|_{Y=\text{const.}} + \frac{\partial C_i}{\partial Y}\frac{\partial Y}{\partial P_L^*} \right)\frac{\partial P_L^*}{\partial P_j}\frac{P_j}{C_i} \\
(116)\qquad &= \left.\frac{\partial C_i}{\partial P_j}\right|_{P_L^*=\text{const.}} \frac{P_j}{C_i} + \frac{\partial C_i^H}{\partial P_L^*}\frac{P_L^*}{C_i}\frac{\partial P_L^*}{\partial P_j}\frac{P_j}{P_L^*} \\
(117)\qquad &= \Theta_{ij}^{sFHM} + \Theta_{iL}^{H}\Psi_j
\end{aligned}
$$

Substituting the direct component, which is the price elasticity of the separable model Θ_{ij}^{sFHM}, we get the elasticities for each of the exogenous prices:

$$
\begin{aligned}
(118)\ \Theta_{ij}^{iFHM} &= \eta_i \frac{P_j X_j}{Y} + \Theta_{iL}^{H}\Psi_j \quad \forall\, i \in \{m,a,L\},\ j \in \{c,v\} \\
(119)\ \Theta_{ia}^{iFHM} &= \Theta_{ia}^{H} + \eta_i \frac{P_a\left(X_a - C_a\right)}{Y} + \Theta_{iL}^{H}\Psi_a \quad \forall\, i \in \{m,a,L\} \\
(120)\ \Theta_{im}^{iFHM} &= \Theta_{im}^{H} - \eta_i \frac{P_m C_m}{Y} + \Theta_{iL}^{H}\Psi_m \quad \forall\, i \in \{m,a,L\}
\end{aligned}
$$

Price Elasticities of Labor Allocation

We derive the price elasticities of labor supply and demand from equation (13) of the main article. Since the labor supply and demand do not directly depend on the exogenous prices, the direct component is zero:

$$
\begin{aligned}
(121)\ \varphi_{sj}^{iFHM} &= \frac{\partial X_L^s}{\partial P_L^*}\frac{P_L^*}{X_L^s}\frac{\partial P_L^*}{\partial P_j}\frac{P_j}{P_L^*} \\
(122)\qquad &= \varphi_L^s \Psi_j \quad \forall\, j \in \{c,a,v,m\} \\
(123)\ \varphi_{hj}^{iFHM} &= \frac{\partial X_L^h}{\partial P_L^*}\frac{P_L^*}{X_L^h}\frac{\partial P_L^*}{\partial P_j}\frac{P_j}{P_L^*} \\
(124)\qquad &= \varphi_L^h \Psi_j \quad \forall\, j \in \{c,a,v,m\}
\end{aligned}
$$

The remaining labor allocation elasticities are calculated residually:

$$(125)\quad \varphi_{nj}^{iFHM} = \frac{\partial\left(X_L^s - X_L^h\right)}{\partial P_L^*}\frac{P_L^*}{X_L^n}\frac{\partial P_L^*}{\partial P_j}\frac{P_j}{P_L^*}$$

$$(126)\quad = \frac{\partial X_L^s}{\partial P_L^*}\frac{P_L^*}{X_L^s}\frac{\partial P_L^*}{\partial P_j}\frac{P_j}{P_L^*}\frac{X_L^s}{X_L^n} - \frac{\partial X_L^h}{\partial P_L^*}\frac{P_L^*}{X_L^h}\frac{\partial P_L^*}{\partial P_j}\frac{P_j}{P_L^*}\frac{X_L^s}{X_L^n}$$

$$(127)\quad = \varphi_j^s\frac{X_L^s}{X_L^n} - \varphi_j^h\frac{X_L^h}{X_L^n}\; j \in \{c,a,v,m\}$$

$$(128)\quad \varphi_{fj}^{iFHM} = \frac{\partial\left(T_L - X_L^s - C_L^h\right)}{\partial P_L^*}\frac{P_L^*}{X_L^f}\frac{\partial P_L^*}{\partial P_j}\frac{P_j}{P_L^*}$$

$$(129)\quad = -\frac{\partial X_L^s}{\partial P_L^*}\frac{P_L^*}{X_L^s}\frac{\partial P_L^*}{\partial P_j}\frac{P_j}{P_L^*}\frac{X_L^s}{X_L^f} - \frac{\partial C_L}{\partial P_L^*}\frac{P_L^*}{C_L}\frac{\partial P_L^*}{\partial P_j}\frac{P_j}{P_L^*}\frac{C_L}{X_L^f}$$

$$(130)\quad = -\varphi_j^s\frac{X_L^s}{X_L^f} - \Theta_{Lj}^{iFHM}\frac{C_L}{X_L^f}\; j \in \{c,a,v,m\}$$

8 Data Description

Table 2. Characteristics of the Sample

Variable	Unit	Mean	Minimum	Maximum	Std.deviation
N_k	number	1.3	0.0	5.0	1.2
N_w	number	2.8	0.0	7.0	1.3
N_o	number	0.7	0.0	3.0	0.8
A_h	years	43	20	76	11
T_L	hours	11399	3650	27375	4457
$\|X_L\|$	hours	3686	400	9843	1717
X_L^h	hours	211	0	2085	365
X_L^s	hours	446	0	4000	876
X_L^n	hours	235	-2085	4000	1002
X_L^f	hours	3475	400	9236	1705
C_L	hours	7478	23	20873	4007
P_mC_m	1000 PLZ	91469	26365	280176	42853
P_aC_a	1000 PLZ	19041	1625	41853	7606
P_cX_c	1000 PLZ	132258	10451	1189412	133724
P_aX_a	1000 PLZ	212570	2669	2526524	239835
$P_v\|X_v\|$	1000 PLZ	211960	13480	2204671	213479
R_g	ha	14.7	1.2	101.5	12.4
R_k	1000 PLZ	649191	43960	4492025	554120
R_k/R_g	1000 PLZ / ha	46921	9170	215652	29039
N_c	number	0.9	0.0	3.0	0.6
W_u	%	19	9	25	4
W_i	km/100 km^2	58	39	71	9
W_t	1/1000 population	48	31	60	9
W_r	%	45	29	58	10
$\widetilde{P}_L$	Poland = 100	88	73	115	13
P_L^*	1000 PLZ/h	38	6	230	28

Note: Calculations are based on IERiGZ (1995). PLZ = Polish Zloty. Variables: N_k = number of family members up to 14 years, N_w = number of family members between 15 and 60 years, N_o = number of family members older than 60 years, A_h = age of the household head, T_L = total time available, $|X_L|$ = labor input on the farm, X_L^h = hired labor, X_L^s = supplied labor, X_L^n = net supplied labor, X_L^f = family labor input on the farm, C_L = leisure, P_mC_m = value of consumed market goods, P_aC_a = value of consumed self-produced goods, P_cX_c = value of produced crop products, P_aX_a = value of produced animal products, $P_v|X_v|$ = value of utilized variable inputs, R_g = amount of land of the farm, R_k = amount of capital of the farm, N_c = number of cars owned by the household, W_u = regional unemployment rate, W_i = regional density of the road and railroad network, W_t = regional density of telephones, W_r = proportion of the population that lives in rural areas, $\widetilde{P}_L$ = relative average regional wage level, $P_.^*$ = endogenous shadow price of labor.

Table 3. Characteristics of the Different Labor Regimes

Variable	Unit	All	Sup. & Dem.	Only Sup.	Only Dem.	Autarkic
Number		199	57	47	61	34
N_k	number	1.3	1.5	1.3	1.4	0.7
N_w	number	2.8	2.8	3.2	2.4	3.0
N_o	number	0.7	0.6	0.6	0.8	0.7
A_h	years	43	41	44	43	45
T_L	hours	11399	11110	12891	10082	12185
$\lvert X_L\rvert$	hours	3686	3579	3372	4040	3668
X_L^h	hours	211	278	0	430	0
X_L^s	hours	446	515	1266	0	0
X_L^n	hours	235	237	1266	-430	0
X_L^f	hours	3475	3301	3372	3610	3668
C_L	hours	7478	7295	8254	6473	8517
P_mC_m	1000 PLZ	91469	105939	78012	97792	74467
P_aC_a	1000 PLZ	19041	18487	19245	19939	18076
P_cX_c	1000 PLZ	132258	157581	65883	180020	95869
P_aX_a	1000 PLZ	212570	220643	123997	300046	164531
$P_v\lvert X_v\rvert$	1000 PLZ	211960	232143	117552	299629	151343
R_g	ha	14.7	16.9	9.4	18.3	11.7
R_k	1000 PLZ	649191	788881	425398	816534	424132
R_k/R_g	1000 PLZ / ha	46921	49666	48516	48134	37938
N_c	number	0.9	1.0	0.8	0.9	0.8
W_u	%	19	20	19	18	20
W_i	km/100 km^2	58	55	60	60	57
W_t	1/1000 popul.	48	47	49	49	47
W_r	%	45	44	50	43	46
$\widetilde{P}_L$	Poland = 100	88	85	90	89	88
P_L^*	1000 PLZ/h	38	46	30	44	28

Note: Calculations are based on IERiGZ (1995). PLZ = Polish Zloty. Variables: N_k = number of family members up to 14 years, N_w = number of family members between 15 and 60 years, N_o = number of family members older than 60 years, A_h = age of the household head, T_L = total time available, $|X_L|$ = labor input on the farm, X_L^h = hired labor, X_L^s = supplied labor, X_L^n = net supplied labor, X_L^f = family labor input on the farm, C_L = leisure, P_mC_m = value of consumed market goods, P_aC_a = value of consumed self-produced goods, P_cX_c = value of produced crop products, P_aX_a = value of produced animal products, $P_v|X_v|$ = value of utilized variable inputs, R_g = amount of land of the farm, R_k = amount of capital of the farm, N_c = number of cars owned by the household, W_u = regional unemployment rate, W_i = regional density of the road and railroad network, W_t = regional density of telephones, W_r = proportion of the population that lives in rural areas, $\widetilde{P}_L$ = relative average regional wage level, $P_\cdot^*$ = endogenous shadow price of labor.

9 Estimation Results

9.1 First-Stage Profit Function

Table 4. Estimation Results of the Unrestricted 1st-Stage Profit Function

Parameter	$i=c$		$i=a$		$i=v$	
	Coef.	(t-val)	Coef.	(t-val)	Coef.	(t-val)
α_i	-1.72	(-0.73)	20.1	(4.31)	-17.4	(-5.14)
β_{ic}	-14.8	(-1.12)	19.8	(2.68)	-4.92	(-0.37)
β_{ia}	19.8	(2.68)	61.6	(5.76)	-81.4	(-8.04)
β_{iv}	-4.92	(-0.37)	-81.4	(-8.04)	86.3	(5.08)
δ_{ig}	6258	(11.37)	1002	(0.93)	-4306	(-5.37)
δ_{ik}	0.0829	(5.77)	0.209	(7.47)	-0.111	(-5.36)
γ_{gg}	-1157392	(-6.45)				
γ_{gk}	36.7	(7.59)				
γ_{kk}	$-1.26{\cdot}10^{-3}$	(-9.79)				
R^2	0.709		0.286		0.685	

Note: For definitions of the estimated coefficients see equation (20) of the main article, where the subscripts c, a, v, g, and k indicate crop products, animal products, variable inputs, land, and capital, respectively. The standard errors of the coefficients that have not been directly estimated are calculated with the formula of Klein (1953, p. 258). Monotonicity is fulfilled at 100% of the observations.

Table 5. Estimation Results of the 1st-Stage Profit Function with Convexity Imposed

Parameter	$i=c$		$i=a$		$i=v$	
	Coef.	(t-val)	Coef.	(t-val)	Coef.	(t-val)
α_i	-2.28	(-0.57)	20.3	(3.16)	-17.0	(-3.21)
β_{ic}	3.31	(0.81)	14.6	(2.34)	-17.9	(-1.99)
β_{ia}	14.6	(2.34)	64.7	(2.93)	-79.3	(-3.16)
β_{iv}	-17.9	(-1.99)	-79.3	(-3.16)	97.3	(3.30)
δ_{ig}	6170	(4.60)	1024	(0.59)	-4294	(-2.26)
δ_{ik}	0.0855	(2.92)	0.208	(4.81)	-0.110	(-3.87)
γ_{gg}	-1149343	(-1.72)				
γ_{gk}	36.6	(1.89)				
γ_{kk}	$-1.26{\cdot}10^{-3}$	(-2.26)				
R^2	0.708		0.283		0.686	

Note: For definitions of the estimated coefficients see equation (20) of the main article, where the subscripts c, a, v, g, and k indicate crop products, animal products, variable inputs, land, and capital, respectively. The standard errors of the coefficients are calculated using the bootstrap resampling method (Efron 1979; Efron and Tibshirani 1993). Monotonicity is fulfilled at 100% of the observations. The R^2 values are almost identical to the model without convexity imposed, indicating that the data do not unreasonably contradict the convexity constraint (see table 4).

Shadow Prices of Labor

One estimated shadow price is negative. The other shadow prices have a mean of 38498 PLZ/h and a median of 30236 PLZ/h. In 1994 the average gross wage in Poland was 32820 PLZ/h. 68% of the estimated shadow prices deviate less than 50% from this value.

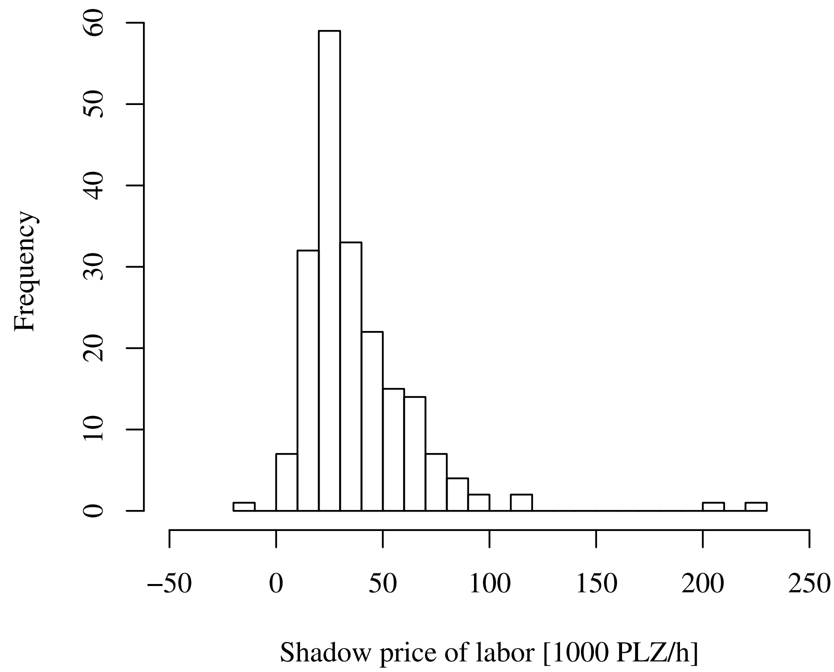


Figure 1. Distribution of the estimated shadow prices of labor

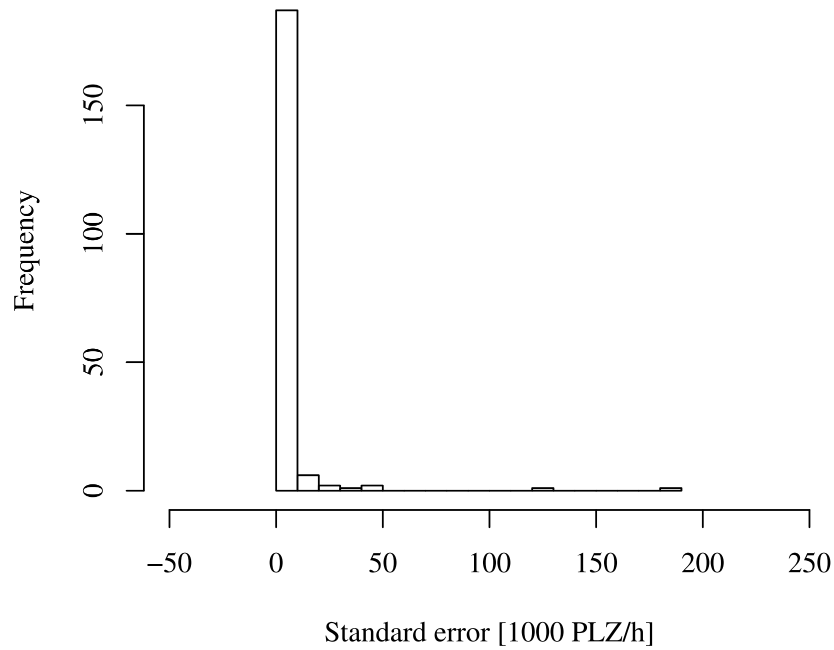
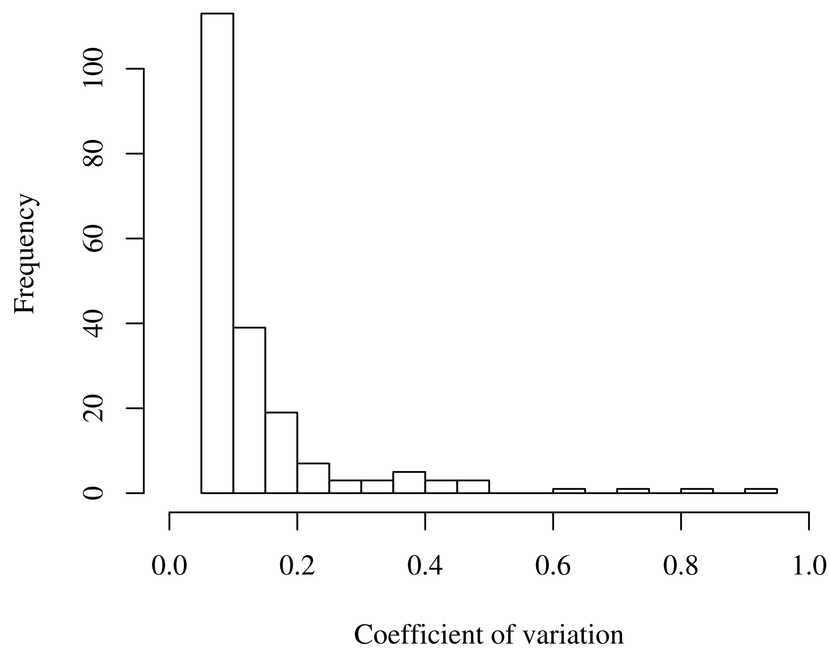


Figure 2. Distribution of the standard errors of the estimated shadow prices of labor



Note: Only coefficients of variation of positive shadow prices are shown.

Figure 3. Coefficients of variation of the estimated shadow prices of labor

9.2 Second-Stage Profit Function

Table 6. Estimation Results of the Unrestricted 2nd-Stage Profit Function

Parameter	$i=c$		$i=a$		$i=v$		$i=L$	
	Coef.	(t-val)	Coef.	(t-val)	Coef.	(t-val)	Coef.	(t-val)
α_i	-28774	(-3.22)	32491	(2.05)	-6714	(-0.57)	-62854	(-12.61)
β_{ic}	879	(0.02)	95377	(2.76)	-61671	(-1.14)	-34585	(-4.22)
β_{ia}	95377	(2.76)	76676	(1.19)	-162987	(-2.97)	-9066	(-0.63)
β_{iv}	-61671	(-1.14)	-162987	(-2.97)	221688	(2.95)	2970	(0.24)
β_{iL}	-34585	(-4.22)	-9066	(-0.63)	2970	(0.24)	40681	(7.48)
δ_{ig}	6896	(11.68)	131	(0.12)	-6000	(-7.02)	-3158	(-8.95)
δ_{ik}	0.121	(9.02)	0.292	(12.21)	-0.166	(-9.31)	$7.41 \cdot 10^{-3}$	(0.93)
γ_{gg}	-173	(-3.55)						
γ_{gk}	$9.88 \cdot 10^{-3}$	(9.24)						
γ_{kk}	$-3.55 \cdot 10^{-7}$	(-24.28)						
R^2	0.746		0.494		0.821		0.283	

Note: For definitions of the estimated coefficients see equation (15) of the main article, where the subscripts c, a, v, L, g, and k indicate crop products, animal products, variable inputs, labor, land, and capital, respectively. The standard errors of the coefficients that have not been directly estimated are calculated with the formula of Klein (1953, p. 258). Monotonicity is fulfilled at 98.0% of the observations. The estimation results with convexity imposed are presented in the main article, table 2.

Table 7. Estimation Results of the 2nd-Stage Profit Function with Convexity Imposed

Parameter	$i=c$		$i=a$		$i=v$		$i=L$	
	Coef.	(t-val)	Coef.	(t-val)	Coef.	(t-val)	Coef.	(t-val)
α_i	-31261	(-2.31)	33699	(2.07)	-5480	(-0.37)	-62939	(-6.95)
β_{ic}	53083	(1.86)	64866	(2.75)	-84580	(-2.13)	-33368	(-3.46)
β_{ia}	64866	(2.75)	116773	(2.47)	-168328	(-2.68)	-13311	(-0.63)
β_{iv}	-84580	(-2.13)	-168328	(-2.68)	247344	(2.72)	5564	(0.32)
β_{iL}	-33368	(-3.46)	-13311	(-0.63)	5564	(0.32)	41115	(6.28)
δ_{ig}	6815	(4.59)	303	(0.14)	-6087	(-4.04)	-3181	(-2.81)
δ_{ik}	0.124	(4.40)	0.291	(7.49)	-0.167	(-6.97)	$7.87 \cdot 10^{-3}$	(0.20)
γ_{gg}	-172	(-1.28)						
γ_{gk}	$9.84 \cdot 10^{-3}$	(2.09)						
γ_{kk}	$-3.55 \cdot 10^{-7}$	(-2.26)						
R^2	0.747		0.492		0.821		0.278	

Note: For definitions of the estimated coefficients see equation (15) of the main article, where the subscripts c, a, v, L, g, and k indicate crop products, animal products, variable inputs, labor, land, and capital, respectively. The standard errors of the coefficients are calculated using the bootstrap resampling method (Efron 1979; Efron and Tibshirani 1993). Monotonicity is fulfilled at 97.0% of the observations. The R^2 values are almost identical to the model without convexity imposed, indicating that the data do not unreasonably contradict the convexity constraint (see table 6).

Table 8. Price Elasticities of the Restricted 2nd-Stage Profit Function

	P_c		P_a		P_v		P_L	
	Coef.	(t-val.)	Coef.	(t-val.)	Coef.	(t-val.)	Coef.	(t-val.)
X_c	0.429	(1.99)	0.503	(2.90)	-0.567	(-2.03)	-0.364	(-3.77)
X_a	0.320	(2.90)	0.533	(2.49)	-0.735	(-2.62)	-0.117	(-0.88)
X_v	0.356	(2.03)	0.726	(2.62)	-1.081	(-2.69)	-0.001	(-0.01)
X_L	0.340	(3.77)	0.172	(0.88)	-0.002	(-0.01)	-0.511	(-6.29)

9.3 AIDS Model

Table 9. Estimation Results of the AIDS

Parameter	$i = m$		$i = a$		$i = L$	
	Coef.	(t-val.)	Coef.	(t-val.)	Coef.	(t-val.)
α_i	0.555	(9.86)	0.185	(14.79)	0.260	(4.18)
β_i	-0.170	(-9.15)	-0.031	(-7.36)	0.201	(9.95)
γ_{im}	0.034	(1.28)	0.021	(0.79)	-0.055	(-5.34)
γ_{ia}	0.021	(0.79)	0.010	(0.35)	-0.031	(-9.36)
γ_{iL}	-0.055	(-5.34)	-0.031	(-9.36)	0.086	(7.97)
R^2	0.409		0.585		0.504	

Note: For definitions of the estimated coefficients see equation (16), where the subscripts m, a, and L indicate purchased market goods, self-produced goods, and leisure, respectively. The standard errors of the coefficients that have not been directly estimated are calculated with the formula of Klein (1953, p. 258). α_0 is set to 10.8, because this value gives the highest likelihood value of the AIDS Model. Monotonicity is fulfilled at 99.5% of the observations and concavity is fulfilled at 88.4% of the observations.

Table 10. Price and Income Elasticities of the AIDS Model

	P_m		P_a		P_L	
	Elast.	(t-val.)	Elast.	(t-val.)	Elast.	(t-val.)
Hicksian Price Elasticities						
C_m	-0.554	(-5.67)	0.144	(1.53)	0.409	(8.59)
C_a	0.648	(1.55)	-0.782	(-1.80)	0.134	(2.55)
C_L	0.176	(8.58)	0.014	(2.77)	-0.190	(-8.53)
Marshallian Price Elasticities						
C_m	-0.667	(-6.80)	0.119	(1.26)	0.149	(2.09)
C_a	0.503	(1.20)	-0.814	(-1.88)	-0.200	(-2.62)
C_L	-0.194	(-9.46)	-0.070	(-13.34)	-1.045	(-31.28)
Income Elasticities						
Y	0.399	(6.08)	0.511	(7.70)	1.308	(42.25)

9.4 Labor Market Estimations

The analysis of labor supply and demand of the households is summarized in table 4 of the main article. The bivariate probit estimation shows that labor demand and supply decisions are not significantly correlated in the sample (ρ is not significantly different from zero). The probability that a household supplies off-farm labor increases significantly with the number of household members of working age (N_w) and with the rural nature of the region (W_r).

The probability that a household demands labor significantly depends on the capital endowment (R_k), the endowment of family labor (N_w, N_o), the age of the head of the household (A_h, A_h^2), and the rural nature of the region (W_r). As expected, the probability increases with the capital endowment and decreases with the endowment of family labor. We also observe the expected signs for the age and squared age of the household head, i.e. we observe a u-shaped relation between age and the probability to hire on-farm labor with the lowest probability at the age of 44.4 years. Furthermore, the probability to hire labor decreases with the rural nature of the region.

The effective off-farm wage is significantly influenced by the proportion of supplied labor (X_L^s/T_L), the number of family members of working age (N_w), the age of the head of the household (A_h, A_h^2), and the rural nature of the region (W_r). Larger households and those in more rural areas receive a significantly lower effective off-farm wage. The coefficients of the age and squared age of the household head have the expected signs; i.e. we observe an inverse u-shaped relation between age and the effective off-farm wage with the highest wage at the age of 44.2 years. The estimated parameter of the inverse Mill's ratio is not significantly different from zero, indicating that there is no sample selection bias. If an average household (see table 2 of the main article) increases the amount of supplied labor by 1%, the marginal revenue decreases by 0.075%. If this household doubles the amount of supplied labor from 446 to 892 hours per year, the marginal revenue decreases from 38498 to 35618 PLZ per hour.

The effective on-farm wage is significantly influenced by the amount of hired labor (X_L^h), the capital intensity on the farm (R_k/R_g), the regional unemployment rate (W_u), the regional density of the road and railroad network (W_i), and the rurality of the region (W_r). As expected, farms with a higher degree of mechanization pay higher wages because better skills are required on these farms. The negative impact of the rural nature and the positive impact of the road and railroad network on the effective on-farm wage might reflect heterogeneity of the average regional wages that is not captured in the regional data published by the statistical office ($\widetilde{P}_L$). The positive effect of the regional unemployment rate is counter-intuitive. However, it might be correlated with some other regional variable not included in the analysis. In contrast to the labor supply side, the estimated parameter of the inverse Mill's ratio is significantly different from zero, indicating that an OLS estimation for labor-hiring households would be biased due to non-random sample selection. If an average household (see table 2 of the main article) increases the amount of hired labor by 1%, the marginal cost increases by 0.259%. If this household doubles the amount of hired labor from 211 to 422 hours per year, the marginal cost increases from 38498 to 48467 PLZ per hour.

10 Estimated Farm-Household Elasticities

10.1 Elasticities for Different Labor Regimes

Table 11. Price Elasticities of the Separable FHM (Calculated at Average Values of All Households)

	P_c		P_a		P_v		P_L		P_m	
	Elast.	(t-val.)	Elast.	(t-val.)	Elast.	(t-val.)	Elast.	(t-val.)	Elast.	(t-val.)
X_c	0.43	(1.99)	0.50	(2.90)	-0.57	(-2.03)	-0.36	(-3.77)	0.00	
X_a	0.32	(2.90)	0.53	(2.49)	-0.73	(-2.62)	-0.12	(-0.88)	0.00	
X_v	0.36	(2.03)	0.73	(2.62)	-1.08	(-2.69)	-0.00	(-0.01)	0.00	
X_L	0.34	(3.77)	0.17	(0.88)	-0.00	(-0.01)	-0.51	(-6.29)	0.00	
C_m	0.13	(6.08)	0.33	(3.26)	-0.21	(-6.08)	0.45	(4.20)	-0.67	(-6.80)
C_a	0.17	(7.70)	-0.55	(-1.25)	-0.27	(-7.70)	0.18	(0.41)	0.50	(1.20)
C_L	0.43	(42.25)	0.61	(39.18)	-0.69	(-42.25)	-0.07	(-3.22)	-0.19	(-9.46)
X_L^n	-19.15	(-13.18)	-22.20	(-7.11)	22.00	(9.08)	10.30	(7.07)	6.16	(9.46)
X_L^f	0.34	(3.77)	0.17	(0.88)	-0.00	(-0.01)	-0.51	(-6.29)	0.00	
P_L^*	0.00		0.00		0.00		1.00		0.00	

Table 12. Price Elasticities of the Non-separable FHM for Households that Both Supply and Hire Labor (Calculated at Average Values of All Households)

	P_c		P_a		P_v		P_m	
	Elast.	(t-val.)	Elast.	(t-val.)	Elast.	(t-val.)	Elast.	(t-val.)
X_c	0.28	(1.51)	0.33	(2.06)	-0.39	(-1.53)	0.05	(2.67)
X_a	0.27	(2.40)	0.48	(2.30)	-0.68	(-2.26)	0.02	(0.87)
X_v	0.36	(2.10)	0.73	(2.57)	-1.08	(-2.61)	0.00	(0.01)
X_L	0.13	(1.43)	-0.08	(-0.50)	0.24	(1.98)	0.07	(3.32)
C_m	0.30	(6.21)	0.53	(4.76)	-0.41	(-6.74)	-0.72	(-7.32)
C_a	0.23	(7.53)	-0.48	(-1.13)	-0.33	(-8.25)	0.48	(1.16)
C_L	0.35	(15.54)	0.52	(18.27)	-0.60	(-21.22)	-0.17	(-7.98)
X_L^h	1.52	(1.46)	1.76	(1.26)	-1.75	(-1.37)	-0.49	(-1.26)
X_L^s	-6.26	(-3.79)	-7.25	(-3.56)	7.19	(3.69)	2.01	(3.55)
X_L^n	-13.25	(-3.46)	-15.37	(-3.42)	15.23	(3.45)	4.26	(3.42)
X_L^f	0.04	(0.16)	-0.19	(-0.60)	0.37	(1.22)	0.10	(1.28)
P_L^*	0.42	(3.94)	0.49	(3.68)	-0.48	(-3.82)	-0.13	(-3.67)

Table 13. Price Elasticities of the Non-separable FHM for Households that Both Supply and Hire Labor (Calculated at Average Values of Households in this Labor Regime)

	P_c		P_a		P_v		P_m	
	Elast.	(t-val.)	Elast.	(t-val.)	Elast.	(t-val.)	Elast.	(t-val.)
X_c	0.28	(1.53)	0.33	(2.09)	-0.40	(-1.55)	0.05	(2.56)
X_a	0.27	(2.44)	0.48	(2.31)	-0.68	(-2.27)	0.02	(0.86)
X_v	0.36	(2.10)	0.73	(2.57)	-1.08	(-2.62)	0.00	(0.01)
X_L	0.14	(1.51)	-0.06	(-0.41)	0.23	(1.84)	0.07	(3.11)
C_m	0.30	(5.87)	0.52	(4.64)	-0.40	(-6.42)	-0.72	(-7.28)
C_a	0.22	(7.50)	-0.49	(-1.13)	-0.33	(-8.20)	0.49	(1.17)
C_L	0.36	(15.23)	0.52	(17.96)	-0.60	(-20.81)	-0.17	(-7.99)
X_L^h	1.30	(1.35)	1.51	(1.18)	-1.50	(-1.27)	-0.42	(-1.18)
X_L^s	-5.47	(-3.46)	-6.34	(-3.28)	6.29	(3.38)	1.76	(3.28)
X_L^n	-13.41	(-3.12)	-15.55	(-3.10)	15.41	(3.11)	4.31	(3.10)
X_L^f	0.04	(0.14)	-0.19	(-0.55)	0.37	(1.12)	0.11	(1.17)
P_L^*	0.40	(3.60)	0.46	(3.39)	-0.46	(-3.51)	-0.13	(-3.40)

Note: To focus on the effect of the labor market regime, only X_L^s, X_L^h, z^s and z^h are the average values of households in this labor regime, while X_c, X_a, X_v, X_L, C_m, X_a and C_L are taken from the whole sample. $X_L^F = X_L - X_L^H$ and $T_L = X_L^S + X_L^F + C_L$ are calculated residually.

Table 14. Price Elasticities of the Non-separable FHM for Households that only Supply Labor (Calculated at Average Values of Households in this Labor Regime)

	P_c		P_a		P_v		P_m	
	Elast.	(t-val.)	Elast.	(t-val.)	Elast.	(t-val.)	Elast.	(t-val.)
X_c	0.24	(1.05)	0.29	(1.23)	-0.35	(-1.16)	0.06	(1.13)
X_a	0.26	(2.06)	0.46	(2.14)	-0.67	(-2.14)	0.02	(0.71)
X_v	0.36	(2.10)	0.73	(2.55)	-1.08	(-2.60)	0.00	(0.01)
X_L	0.08	(0.35)	-0.13	(-0.46)	0.30	(1.10)	0.08	(1.17)
C_m	0.34	(1.93)	0.57	(2.52)	-0.45	(-2.20)	-0.73	(-6.51)
C_a	0.24	(3.69)	-0.47	(-1.09)	-0.35	(-4.52)	0.48	(1.16)
C_L	0.34	(4.09)	0.50	(5.20)	-0.58	(-6.08)	-0.16	(-4.94)
X_L^s	-2.22	(-1.16)	-2.58	(-1.15)	2.55	(1.16)	0.72	(1.15)
X_L^f	0.08	(0.10)	-0.13	(-0.14)	0.30	(0.32)	0.08	(0.32)
P_L^*	0.51	(1.19)	0.59	(1.18)	-0.59	(-1.19)	-0.16	(-1.18)

Note: see note below table 13.

Table 15. Price Elasticities of the Non-separable FHM for Households that only Hire Labor (Calculated at Average Values of Households in this Labor Regime)

	P_c		P_a		P_v		P_m	
	Elast.	(t-val.)	Elast.	(t-val.)	Elast.	(t-val.)	Elast.	(t-val.)
X_c	0.05	(0.33)	0.06	(0.41)	-0.13	(-0.56)	0.12	(3.32)
X_a	0.20	(1.28)	0.39	(1.65)	-0.59	(-1.69)	0.04	(0.91)
X_v	0.36	(1.97)	0.72	(2.37)	-1.08	(-2.46)	0.00	(0.01)
X_L	-0.19	(-2.56)	-0.45	(-4.59)	0.61	(6.94)	0.17	(5.35)
C_m	0.56	(9.20)	0.82	(7.10)	-0.70	(-9.84)	-0.80	(-8.25)
C_a	0.31	(5.71)	-0.38	(-0.92)	-0.43	(-6.70)	0.46	(1.11)
C_L	0.23	(8.30)	0.38	(10.99)	-0.46	(-13.92)	-0.13	(-6.62)
X_L^h	2.42	(0.40)	2.80	(0.40)	-2.78	(-0.40)	-0.78	(-0.40)
X_L^f	-0.54	(-8.30)	-0.88	(-10.99)	1.06	(13.92)	0.30	(6.62)
P_L^*	1.05	(8.01)	1.21	(7.39)	-1.20	(-7.92)	-0.34	(-5.61)

Note: see note below table 13.

Table 16. Price Elasticities of the Non-separable FHM for Autarkic Households (Calculated at Average Values of Households in this Labor Regime)

	P_c		P_a		P_v		P_m	
	Elast.	(t-val.)	Elast.	(t-val.)	Elast.	(t-val.)	Elast.	(t-val.)
X_c	-0.07	(-0.51)	-0.07	(-0.49)	0.00	(0.01)	0.16	(3.40)
X_a	0.16	(0.88)	0.35	(1.32)	-0.55	(-1.44)	0.05	(0.92)
X_v	0.35	(1.84)	0.72	(2.24)	-1.08	(-2.37)	0.00	(0.01)
X_L	-0.35	(-5.77)	-0.63	(-9.15)	0.80	(11.88)	0.22	(6.00)
C_m	0.69	(10.50)	0.97	(8.51)	-0.85	(-11.88)	-0.85	(-8.75)
C_a	0.35	(5.16)	-0.34	(-0.82)	-0.48	(-6.07)	0.44	(1.08)
C_L	0.17	(5.77)	0.31	(9.15)	-0.39	(-11.88)	-0.11	(-6.00)
X_L^f	-0.35	(-5.77)	-0.63	(-9.15)	0.80	(11.88)	0.22	(6.00)
P_L^*	1.36	(9.17)	1.58	(9.44)	-1.56	(-9.79)	-0.44	(-5.65)

Note: see note below table 13.

Table 17. Differences between Price Elasticities of the Separable FHM and the Non-separable FHM for Households that Supply as well as Demand Labor (Calculated at Average Values of All Households)

	P_c		P_a		P_v		P_m	
	Diff.	(t-val.)	Diff.	(t-val.)	Diff.	(t-val.)	Diff.	(t-val.)
X_c	0.15	(2.43)	0.18	(2.73)	-0.18	(-2.55)	-0.05	(-2.67)
X_a	0.05	(0.87)	0.06	(0.78)	-0.06	(-0.81)	-0.02	(-0.87)
X_v	0.00	(0.01)	0.00	(0.01)	-0.00	(-0.01)	-0.00	(-0.01)
X_L	0.21	(3.37)	0.25	(2.85)	-0.25	(-3.08)	-0.07	(-3.32)
C_m	-0.17	(-3.64)	-0.20	(-3.42)	0.20	(3.54)	0.06	(4.02)
C_a	-0.06	(-2.16)	-0.07	(-2.09)	0.06	(2.14)	0.02	(2.19)
C_L	0.08	(3.64)	0.09	(3.41)	-0.09	(-3.54)	-0.03	(-4.00)
X_L^n	-5.90	(-1.50)	-6.84	(-1.38)	6.77	(1.44)	1.90	(1.47)
X_L^f	0.30	(1.21)	0.36	(1.24)	-0.37	(-1.28)	-0.10	(-1.28)
P_L^*	-0.42	(-3.94)	-0.49	(-3.68)	0.48	(3.82)	0.13	(3.67)

Table 18. Differences between Price Elasticities of the Separable FHM and the Non-separable FHM for Households that Supply as well as Demand Labor

	P_c		P_a		P_v		P_m	
	Diff.	(t-val.)	Diff.	(t-val.)	Diff.	(t-val.)	Diff.	(t-val.)
X_c	0.15	(2.34)	0.17	(2.60)	-0.17	(-2.45)	-0.05	(-2.56)
X_a	0.05	(0.87)	0.05	(0.78)	-0.05	(-0.80)	-0.02	(-0.86)
X_v	0.00	(0.01)	0.00	(0.01)	-0.00	(-0.01)	-0.00	(-0.01)
X_L	0.20	(3.14)	0.24	(2.71)	-0.23	(-2.90)	-0.07	(-3.11)
C_m	-0.16	(-3.37)	-0.19	(-3.19)	0.19	(3.29)	0.05	(3.66)
C_a	-0.05	(-2.10)	-0.06	(-2.03)	0.06	(2.08)	0.02	(2.13)
C_L	0.08	(3.37)	0.09	(3.18)	-0.09	(-3.29)	-0.02	(-3.65)
X_L^n	-5.74	(-1.30)	-6.66	(-1.21)	6.60	(1.26)	1.85	(1.28)
X_L^f	0.30	(1.08)	0.37	(1.13)	-0.38	(-1.17)	-0.11	(-1.17)
P_L^*	-0.40	(-3.60)	-0.46	(-3.39)	0.46	(3.51)	0.13	(3.40)

Table 19. Differences between Price Elasticities of the Separable FHM and the Non-separable FHM for Households that only Supply Labor

	P_c		P_a		P_v		P_m	
	Diff.	(t-val.)	Diff.	(t-val.)	Diff.	(t-val.)	Diff.	(t-val.)
X_c	0.19	(1.11)	0.22	(1.14)	-0.21	(-1.12)	-0.06	(-1.13)
X_a	0.06	(0.72)	0.07	(0.66)	-0.07	(-0.68)	-0.02	(-0.71)
X_v	0.00	(0.01)	0.00	(0.01)	-0.00	(-0.01)	-0.00	(-0.01)
X_L	0.26	(1.17)	0.30	(1.15)	-0.30	(-1.16)	-0.08	(-1.17)
C_m	-0.21	(-1.18)	-0.24	(-1.17)	0.24	(1.18)	0.07	(1.19)
C_a	-0.07	(-1.08)	-0.08	(-1.07)	0.08	(1.08)	0.02	(1.08)
C_L	0.10	(1.18)	0.11	(1.17)	-0.11	(-1.18)	-0.03	(-1.19)
X_L^n	-16.93	(-7.30)	-19.62	(-5.46)	19.45	(6.26)	5.45	(6.53)
X_L^f	0.26	(0.32)	0.30	(0.32)	-0.30	(-0.32)	-0.08	(-0.32)
P_L^*	-0.51	(-1.19)	-0.59	(-1.18)	0.59	(1.19)	0.16	(1.18)

Table 20. Differences between Price Elasticities of the Separable FHM and the Non-separable FHM for Households that only Demand Labor

	P_c		P_a		P_v		P_m	
	Diff.	(t-val.)	Diff.	(t-val.)	Diff.	(t-val.)	Diff.	(t-val.)
X_c	0.38	(2.97)	0.44	(3.73)	-0.44	(-3.26)	-0.12	(-3.32)
X_a	0.12	(0.92)	0.14	(0.81)	-0.14	(-0.84)	-0.04	(-0.91)
X_v	0.00	(0.01)	0.00	(0.01)	-0.00	(-0.01)	-0.00	(-0.01)
X_L	0.53	(6.19)	0.62	(4.30)	-0.61	(-5.01)	-0.17	(-5.35)
C_m	-0.43	(-6.86)	-0.50	(-6.38)	0.49	(6.75)	0.14	(8.39)
C_a	-0.14	(-2.52)	-0.16	(-2.45)	0.16	(2.51)	0.05	(2.52)
C_L	0.20	(6.88)	0.23	(6.35)	-0.23	(-6.76)	-0.06	(-8.27)
X_L^n	-21.57	(-3.43)	-25.00	(-3.23)	24.78	(3.34)	6.94	(3.35)
X_L^f	0.88	(9.79)	1.05	(6.42)	-1.06	(-7.87)	-0.30	(-6.62)
P_L^*	-1.05	(-8.01)	-1.21	(-7.39)	1.20	(7.92)	0.34	(5.61)

Table 21. Differences between Price Elasticities of the Separable FHM and the Non-separable FHM for Autarkic Households

	P_c		P_a		P_v		P_m	
	Diff.	(t-val.)	Diff.	(t-val.)	Diff.	(t-val.)	Diff.	(t-val.)
X_c	0.50	(3.06)	0.57	(4.04)	-0.57	(-3.43)	-0.16	(-3.40)
X_a	0.16	(0.93)	0.19	(0.82)	-0.18	(-0.85)	-0.05	(-0.92)
X_v	0.00	(0.01)	0.00	(0.01)	-0.00	(-0.01)	-0.00	(-0.01)
X_L	0.69	(7.97)	0.81	(4.99)	-0.80	(-6.00)	-0.22	(-6.00)
C_m	-0.56	(-8.19)	-0.65	(-8.22)	0.64	(8.52)	0.18	(9.49)
C_a	-0.18	(-2.58)	-0.21	(-2.54)	0.21	(2.59)	0.06	(2.55)
C_L	0.26	(8.24)	0.30	(8.17)	-0.30	(-8.57)	-0.08	(-9.36)
X_L^n	-19.15	(-13.18)	-22.20	(-7.11)	22.00	(9.08)	6.16	(9.46)
X_L^f	0.69	(7.97)	0.81	(4.99)	-0.80	(-6.00)	-0.22	(-6.00)
P_L^*	-1.36	(-9.17)	-1.58	(-9.44)	1.56	(9.79)	0.44	(5.65)

Table 22. Differences between Price Elasticities of the Households that Supply as well as Demand Labor and the Households that only Supply Labor

	P_c		P_a		P_v		P_m	
	Diff.	(t-val.)	Diff.	(t-val.)	Diff.	(t-val.)	Diff.	(t-val.)
X_c	0.04	(0.34)	0.05	(0.34)	-0.05	(-0.34)	-0.01	(-0.34)
X_a	0.01	(0.32)	0.01	(0.32)	-0.01	(-0.32)	-0.00	(-0.32)
X_v	0.00	(0.01)	0.00	(0.01)	-0.00	(-0.01)	-0.00	(-0.01)
X_L	0.06	(0.34)	0.07	(0.34)	-0.06	(-0.34)	-0.02	(-0.34)
C_m	-0.05	(-0.34)	-0.05	(-0.34)	0.05	(0.34)	0.01	(0.34)
C_a	-0.01	(-0.34)	-0.02	(-0.34)	0.02	(0.34)	0.00	(0.34)
C_L	0.02	(0.34)	0.02	(0.34)	-0.02	(-0.34)	-0.01	(-0.34)
X_L^h	1.30	(0.47)	1.51	(0.47)	-1.50	(-0.47)	-0.42	(-0.47)
X_L^s	-3.25	(-6.98)	-3.77	(-5.67)	3.73	(6.32)	1.05	(5.84)
X_L^n	-11.19	(-4.70)	-12.97	(-4.65)	12.85	(4.68)	3.60	(4.66)
X_L^f	-0.04	(-0.07)	-0.06	(-0.10)	0.08	(0.12)	0.02	(0.12)
P_L^*	-0.11	(-0.34)	-0.13	(-0.34)	0.13	(0.34)	0.04	(0.34)

Table 23. Differences between Price Elasticities of the Households that Supply as well as Demand Labor and the Households that only Demand Labor

	P_c		P_a		P_v		P_m	
	Diff.	(t-val.)	Diff.	(t-val.)	Diff.	(t-val.)	Diff.	(t-val.)
X_c	0.24	(2.59)	0.27	(3.12)	-0.27	(-2.81)	-0.08	(-2.77)
X_a	0.08	(0.92)	0.09	(0.81)	-0.09	(-0.84)	-0.02	(-0.91)
X_v	0.00	(0.01)	0.00	(0.01)	-0.00	(-0.01)	-0.00	(-0.01)
X_L	0.33	(4.17)	0.38	(3.52)	-0.38	(-3.81)	-0.11	(-3.77)
C_m	-0.26	(-4.17)	-0.31	(-4.20)	0.30	(4.23)	0.09	(4.28)
C_a	-0.09	(-2.28)	-0.10	(-2.25)	0.10	(2.29)	0.03	(2.25)
C_L	0.12	(4.17)	0.14	(4.20)	-0.14	(-4.23)	-0.04	(-4.27)
X_L^h	-1.11	(-0.21)	-1.29	(-0.21)	1.28	(0.21)	0.36	(0.21)
X_L^s	-5.47	(-3.46)	-6.34	(-3.28)	6.29	(3.38)	1.76	(3.28)
X_L^n	-15.83	(-1.53)	-18.35	(-1.53)	18.18	(1.53)	5.09	(1.53)
X_L^f	0.58	(2.13)	0.68	(2.11)	-0.69	(-2.17)	-0.19	(-2.11)
P_L^*	-0.65	(-4.25)	-0.75	(-4.31)	0.74	(4.33)	0.21	(3.64)

Table 24. Differences between Price Elasticities of the Households that Supply as well as Demand Labor and the Autarkic Households

	P_c		P_a		P_v		P_m	
	Diff.	(t-val.)	Diff.	(t-val.)	Diff.	(t-val.)	Diff.	(t-val.)
X_c	0.35	(2.88)	0.41	(3.76)	-0.40	(-3.22)	-0.11	(-3.09)
X_a	0.11	(0.94)	0.13	(0.83)	-0.13	(-0.86)	-0.04	(-0.93)
X_v	0.00	(0.01)	0.00	(0.01)	-0.00	(-0.01)	-0.00	(-0.01)
X_L	0.49	(6.25)	0.57	(4.68)	-0.56	(-5.32)	-0.16	(-4.89)
C_m	-0.39	(-5.79)	-0.46	(-6.27)	0.45	(6.15)	0.13	(5.76)
C_a	-0.13	(-2.47)	-0.15	(-2.47)	0.15	(2.50)	0.04	(2.42)
C_L	0.18	(5.82)	0.21	(6.26)	-0.21	(-6.19)	-0.06	(-5.74)
X_L^h	1.30	(1.35)	1.51	(1.18)	-1.50	(-1.27)	-0.42	(-1.18)
X_L^s	-5.47	(-3.46)	-6.34	(-3.28)	6.29	(3.38)	1.76	(3.28)
X_L^n	-13.41	(-3.12)	-15.55	(-3.10)	15.41	(3.11)	4.31	(3.10)
X_L^f	0.40	(1.41)	0.44	(1.32)	-0.42	(-1.30)	-0.12	(-1.30)
P_L^*	-0.96	(-5.76)	-1.11	(-6.30)	1.10	(6.15)	0.31	(4.32)

Table 25. Differences between Price Elasticities of the Households that only Supply Labor and the Households that only Demand Labor

	P_c		P_a		P_v		P_m	
	Diff.	(t-val.)	Diff.	(t-val.)	Diff.	(t-val.)	Diff.	(t-val.)
X_c	0.20	(1.14)	0.23	(1.18)	-0.22	(-1.16)	-0.06	(-1.16)
X_a	0.06	(0.75)	0.07	(0.69)	-0.07	(-0.71)	-0.02	(-0.74)
X_v	0.00	(0.01)	0.00	(0.01)	-0.00	(-0.01)	-0.00	(-0.01)
X_L	0.27	(1.22)	0.32	(1.20)	-0.31	(-1.21)	-0.09	(-1.21)
C_m	-0.22	(-1.22)	-0.25	(-1.22)	0.25	(1.22)	0.07	(1.22)
C_a	-0.07	(-1.11)	-0.08	(-1.11)	0.08	(1.11)	0.02	(1.11)
C_L	0.10	(1.22)	0.12	(1.22)	-0.12	(-1.22)	-0.03	(-1.22)
X_L^h	-2.42	(-0.30)	-2.80	(-0.30)	2.78	(0.30)	0.78	(0.30)
X_L^s	-2.22	(-1.16)	-2.58	(-1.15)	2.55	(1.16)	0.72	(1.15)
X_L^n	-4.64	(-0.58)	-5.38	(-0.58)	5.33	(0.58)	1.49	(0.58)
X_L^f	0.62	(0.76)	0.75	(0.79)	-0.76	(-0.82)	-0.21	(-0.81)
P_L^*	-0.54	(-1.22)	-0.62	(-1.22)	0.62	(1.22)	0.17	(1.20)

Table 26. Differences between Price Elasticities of the Households that only Supply Labor and the Autarkic Households

	P_c		P_a		P_v		P_m	
	Diff.	(t-val.)	Diff.	(t-val.)	Diff.	(t-val.)	Diff.	(t-val.)
X_c	0.31	(1.67)	0.36	(1.80)	-0.36	(-1.73)	-0.10	(-1.70)
X_a	0.10	(0.86)	0.12	(0.77)	-0.11	(-0.80)	-0.03	(-0.85)
X_v	0.00	(0.01)	0.00	(0.01)	-0.00	(-0.01)	-0.00	(-0.01)
X_L	0.43	(1.94)	0.50	(1.88)	-0.50	(-1.91)	-0.14	(-1.88)
C_m	-0.35	(-1.92)	-0.40	(-1.94)	0.40	(1.94)	0.11	(1.92)
C_a	-0.11	(-1.57)	-0.13	(-1.58)	0.13	(1.58)	0.04	(1.56)
C_L	0.16	(1.93)	0.19	(1.94)	-0.19	(-1.94)	-0.05	(-1.92)
X_L^s	-2.22	(-1.16)	-2.58	(-1.15)	2.55	(1.16)	0.72	(1.15)
X_L^n	-2.22	(-1.16)	-2.58	(-1.15)	2.55	(1.16)	0.72	(1.15)
X_L^f	0.43	(0.53)	0.50	(0.53)	-0.50	(-0.53)	-0.14	(-0.53)
P_L^*	-0.85	(-1.92)	-0.99	(-1.94)	0.98	(1.94)	0.27	(1.84)

Table 27. Differences between Price Elasticities of the Households that only Demand Labor and the Autarkic Households

	P_c		P_a		P_v		P_m	
	Diff.	(t-val.)	Diff.	(t-val.)	Diff.	(t-val.)	Diff.	(t-val.)
X_c	0.11	(2.28)	0.13	(2.72)	-0.13	(-2.47)	-0.04	(-2.34)
X_a	0.04	(0.94)	0.04	(0.83)	-0.04	(-0.86)	-0.01	(-0.92)
X_v	0.00	(0.01)	0.00	(0.01)	-0.00	(-0.01)	-0.00	(-0.01)
X_L	0.16	(3.32)	0.19	(3.10)	-0.18	(-3.22)	-0.05	(-2.97)
C_m	-0.13	(-3.11)	-0.15	(-3.30)	0.15	(3.23)	0.04	(3.01)
C_a	-0.04	(-2.07)	-0.05	(-2.09)	0.05	(2.10)	0.01	(2.01)
C_L	0.06	(3.12)	0.07	(3.30)	-0.07	(-3.23)	-0.02	(-3.01)
X_L^h	2.42	(0.40)	2.80	(0.40)	-2.78	(-0.40)	-0.78	(-0.40)
X_L^n	2.42	(0.40)	2.80	(0.40)	-2.78	(-0.40)	-0.78	(-0.40)
X_L^f	-0.18	(-4.46)	-0.24	(-5.22)	0.26	(5.63)	0.07	(4.74)
P_L^*	-0.31	(-3.03)	-0.36	(-3.21)	0.36	(3.14)	0.10	(2.68)

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