Colorimetry for CRT displays

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We analyze the sources of error in specifying color in CRT displays. These include errors inherent in the use of the color matching functions of the CIE 1931 standard observer when only colorimetric, not radiometric, calibrations are available. We provide transformation coefficients that prove to correct the deficiencies of this observer very well. We consider four different candidate sets of cone sensitivities. Some of these differ substantially; variation among candidate cone sensitivities exceeds the variation among phosphors. Finally, the effects of the recognized forms of observer variation on the visual responses (cone excitations or cone contrasts) generated by CRT stimuli are investigated and quantitatively specified. Cone pigment polymorphism gives rise to variation of a few per cent in relative excitation by the different phosphors—a variation larger than the errors ensuing from the adoption of the CIE standard observer, though smaller than the differences between some candidate cone sensitivities. Macular pigmentation has a larger influence, affecting mainly responses to the blue phosphor. The estimated combined effect of all sources of observer variation is comparable in magnitude with the largest differences between competing cone sensitivity estimates but is not enough to disrupt very seriously the relation between the L and M cone weights and the isoluminance settings of individual observers. It is also comparable with typical instrumental colorimetric errors, but we discuss these only briefly.

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1. INTRODUCTION

In this paper we show how stimuli from a CRT color display can best be characterized by a user who lacks radiometric measurements and must rely (or prefers to rely) instead on colorimetric data that are based on inaccurate color matching functions, in particular the (X, Y, Z) tristimulus values of the CIE 1931 standard observer and the associated (x, y) chromaticity coordinates. We show that even this somewhat discredited colorimetric standard suffices for very accurate representation of CRT stimuli, not only relative to one another but in absolute terms, in the sense, for example, of specifying the cone contrast that CRT stimuli of different colors will form for a given class of cone. When the standard observer is adopted by using corrective transformation equations that we provide, the resulting error proves to be much smaller than two other sources of error that we also investigate and quantify: the expected variation among real observers and the differences between competing recently proposed standard observers. It is also small by comparison with typical instrumental colorimetric errors (which we discuss only briefly in Section 8).

The tristimulus values of a color stimulus, such as the (X, Y, Z) values for the CIE standard observer, are integrals of the stimulus spectral power distribution, weighted by the color matching functions $\bar{x}(\lambda)$, $\bar{y}(\lambda)$, and $\bar{z}(\lambda)$, which are themselves simply the tristimulus values for an equal-energy spectrum (p. 130 of Ref. 4). Inaccuracy of the color matching functions is directly reflected in the tristimulus values for any stimulus and in turn affects the estimated luminance and chromaticity coordinates.

The inaccuracy of the CIE 1931 standard observer's color matching functions is well known and arises mainly from luminance errors, which are as great as a factor of 4 at short wavelengths.[1] Much better estimates of the color matching functions have long been available. Notable among these are Judd's 1951[2] and Vos's 1978[3] modifications of the 1931 observer, as well as the 1964 CIE large-field observer.[4,5] Naturally, therefore, all currently favored estimates of the long-wavelength-sensitive, midspectral, and short-wavelength-sensitive (L, M, and S) cone spectral sensitivities on which color depends are based on these relatively recent color matching functions. The cone sensitivities of Smith and Pokorny,[6] hereinafter referred to as SP, are based on the Judd functions, here denoted by (X_J, Y_J, Z_J); those of Stockman, MacLeod, and Johnson[1] (SMJ) are based on the CIE 1964 functions for a 10-deg field (X_{10}, Y_{10}, Z_{10}); and those of Stockman and Sharpe[7] (SS) are based on the Stiles–Burch 1959 data[5] that form part of the support for the CIE 1964 functions.

Yet the original CIE 1931 functions are still the basis of nearly all colorimetric measurements. This creates a problem: Most users of color displays must base their color calibration on colorimetric data that are founded on inaccurate assumptions. Here we examine the consequences of error in the CIE functions. We show how

those consequences can be minimized (and, indeed, rendered practically negligible) in colorimetric CRT calibration.

If CIE 1931 (X, Y, Z) colorimetric data are simply treated as equivalent to the later modified (X, Y, Z) systems cited, large errors can result. As noted, discrepancies in derived luminance values will then exceed a factor of 4 at short wavelengths. The L, M and S cone excitations are each defined as a particular linear combination of the tristimulus values on which they are properly based; when the incorrect 1931 tristimulus values are used instead of the proper modified ones, these cone excitations are individually subject to an error comparable with the error for luminance. For CRT phosphors, the errors are far less than for monochromatic lights—indeed they might be considered reassuringly small—but they are still enough to make a difference in exact work. If for example, the L, and M, and S cone excitations (which we will denote here simply by L, M, and S) are obtained by applying the well-known equations of Smith and Pokorny[6] (cf. p. 615 of Ref. 4) to the 1931 tristimulus values instead of to the Judd-modified values for which they were intended (reputedly a common, though nowhere explicitly endorsed or critically examined, practice among vision researchers), the following errors (fortunately not very large) result. For a typical blue CRT phosphor L and M are underestimated relative to S by 9% and 3%, respectively. For the red phosphor, M is underestimated by 2% and L by 1%. For the green phosphor, cone excitations are accurate to within 0.5%.

Greater errors, approaching a 35% underestimation in L and M cone excitation by the blue phosphor, naturally occur if the equations of Stockman *et al.*, expressing cone excitations for a 10-deg field in terms of the CIE 1964 (X_{10}, Y_{10}, Z_{10}) tristimulus values, are applied instead to the 1931 values. Unless these errors can be tolerated, the linear equations given by Smith and Pokorny[6] or by Stockman *et al.*,[1] do, respectively, require Judd-modified or CIE 1964 tristimulus values instead of the CIE 1931 values delivered by colorimetric measurement.

Nevertheless, a different set of three linear equations relating L, M, and S to X, Y, and Z can accommodate the CIE 1931 values in CRT color calibration: The nine degrees of freedom allowed by free choice of the nine coefficients in such a set of equations can be used to correct the cone excitations for each of the three CRT phosphors. The linear relation between L, M, and S and the red, green, and blue phosphor energies R, G, and B guarantees that the resulting transformation is correct not only for each of the three phosphors but also for any CRT stimuli obtained by mixing the phosphors. And this is true regardless of the accuracy of the tristimulus values themselves.

But what is this correct transformation? It depends on the spectral power distribution of the phosphors. When this is known, the problem we are here concerned with can be settled.[8] But how can the appropriate transformation relating L, M, and S to CIE 1931 X, Y, and Z be found in the absence of radiometric data?

Fortunately, CRT phosphor spectra usually do not deviate very much from a typical form. This suggests a very simple solution to the problem. To the extent that the phosphor spectra are similar on different CRTs, the linear transformation from X, Y, Z to L, M, S derived for a *typical* set of red, green, and blue phosphor spectra should be approximately valid for *any* monitor. This approximation does not strictly require that variation among phosphors from one CRT to another be minimal. Any variation in phosphor spectra within each phosphor color category is taken into account, just to the extent that the X, Y, and Z color matching functions indicate its effects correctly. We show here that this approach is effective.

If XYZ and LMS are column vectors, we can represent the transformation from X, Y, Z values to L, M, S values for CRT stimuli as

$$LMS = LMS\_XYZ * XYZ . \tag{1}$$

Each row of the 3×3 matrix LMS_XYZ contains the three coefficients expressing some cone excitation value (L, M, or S) as a weighted sum of the measured tristimulus values X, Y, and Z, which are together determined by the intensities of the three phosphors. Values for this matrix are suggested below. With LMS_XYZ in hand, one can easily obtain cone excitations for any stimulus displayed on that CRT by using Eq. (1). The values of X, Y, and Z are obtained directly by colorimetric measurement of the CRT stimulus of interest.

We have derived the matrix LMS_XYZ for CRT color calibration where the tristimulus vector XYZ is based on the (assumed invalid) CIE 1931 color matching functions (for formulas see Appendix A). Four variants are given for different choices of the cone-sensitivity functions that are taken to be valid:

(a) SP: The cone spectral sensitivities of Smith and Pokorny[6] are based on the Judd[2] functions.

(b) SMJ2: The cone sensitivities of Stockman, MacLeod, and Johnson[1] for a 2-deg field are based on the CIE 1964 functions for a 10-deg field, with corrections for the effects of macular pigmentation and visual pigment density changes associated with the smaller field size. Here we use not the version of the SMJ2 sensitivities fully tabulated in their Table 8 but the one specified in their Table 6D. These sensitivities are preferred here because the 10-deg L and M functions from which they are derived sum exactly to $\bar{y}_{10}(\lambda)$, with weights constrained so that the white point in the constant-luminance chromaticity diagram lies at $l = L/(L+M) = 0.700$ exactly. These L and M sensitivities are similar to those of SMJ's Table 8, and the S sensitivity is identical.

(c) SMJ10: In the same source as SMJ2, cone sensitivities for a 10-deg field are also given; these are a linear combination of the CIE 1964 functions, except that the long-wavelength (>520 nm) S sensitivity is derived independently, since in this range the S sensitivity based on the color matching functions is strongly contaminated by rod intrusion.

(d) SS: The cone sensitivities of Stockman and Sharpe[7] for a 2-deg field are derived in a way broadly similar to the derivation of the SMJ2 functions but are based on the Stiles–Burch 1959 data[5] that form part of the support for the CIE 1964 functions.

In addition, we briefly present other colorimetric data relevant for CRT calibration. Matrices for conversion of

CIE 1931 to Judd-modified or to CIE 1964 *XYZ* values are given for CRT stimuli. Finally, for those interested in estimating cone excitations from CRT stimuli very roughly without doing any colorimetry at all, we give transformations to L, M, and S from red, green, and blue phosphor amounts R, G and B, each measured by their CIE 1931 photometric luminance.

Section 4 specifies the above transformations for typical phosphors, and Section 5 reports that reassuringly small errors ensue from their use with unknown phosphors. Section 6 evaluates the differences in cone excitation by CRT stimuli for different current candidate cone-sensitivity estimates. Section 7 evaluates the influence of known sources of observer variation on cone excitations from CRT stimuli. Section 8 exemplifies typical instrumental colorimetric errors.

2. METHODS

A. Measurements

Spectral data on which the transformation was based were derived from radiometric measurements, 380–780 nm on a haphazard collection of 15 color monitors of different models from eight different manufacturers (Tektronix, Mitsubishi, Nanao/Eizo, Sony, Apple, NEC, Nokia, and Micron Electronics). One additional monitor was rejected because of unusually severe mixing of the phosphor emissions in the output from one gun. The other monitors differed little in this respect, but an indication of the prevalent extent of cross talk is apparent in the average spectra, where the narrow spectral peaks characteristic of the red phosphor are reduced only to 5% for the blue gun and to 0.6% for the green gun when the stimuli from the three guns are equated in luminance. Since this amount of cross talk is doubtless fairly typical, we made no attempt to remove it in our computations. Thus our references to "typical phosphors" more strictly refer to the stimuli that are typically generated by a single gun.

In most cases a Photo Research PR650 spectroradiometer was used, though in a few cases a PR704 was available. The PR650 measures spectral power distributions at 4-nm intervals, with a roughly trapezoidal weighting function of full width 8 nm, originating from the combination of a 4-nm slit with a 4-nm detector extent at the spectrum plane. Checks with He–Ne and Ar laser sources and against other radiometers indicated that the wavelength calibration of the PR650 was accurate to within 1 nm. The R, G, and B components were measured separately at the half-maximum intensity level in order to avoid loss of saturation due to blooming. The spectral power distributions were interpolated as necessary (by use of cubic splines in MATLAB) and averaged.

The spectral resolution of the PR650 or other instruments of this type might appear inadequate for the task at hand, since convolution with the relatively broad instrumental spread function precludes a faithful rendering of the steepest gradients in the phosphor spectra. Calculation indicates that this concern is unwarranted. It is difficult to allow directly for the effects of convolution with an instrumental spectral spread function, since deconvolution introduces large artifacts originating largely from measurement noise. Instead, we adopted the following procedure. The effects of candidate spread functions were evaluated by convolving each spread function ten times in succession with the measured spectral power distributions. The factor by which convolution changed the cone excitations was constant within 10^{-4} over the ten successive convolutions. The initial instrumental convolution implicit in the initial spectral power distributions would therefore doubtless have had an effect similar to the computed effects of the subsequent convolutions with a similar kernel. We estimated the effect of the approximately 6-nm slit width of the PR650 by successive convolutions of the spectra with a 6-nm-wide rectangular spread function or with a 4-nm-to-8-nm trapezoid. This yielded changes so small ($<0.2\%$ in the computed cone excitations) that we did not consider it necessary to make corrections.

B. Units for *L*, *M*, and *S*

The values constituting *LMS_XYZ* depend critically on the choice of particular units for L, M, and S. Only the relative sizes of the units are significant here; the absolute sizes of all three together affect only luminance and not color specification, and an appropriate scaling factor for all values may be obtained from a luminance measurement.

Since L and M cones both contribute to luminance, it is possible and natural to express the L and M cone excitations in luminance units. This convention is adopted in the SP sensitivities and SMJ sensitivities considered here (with the unit for S necessarily somewhat arbitrary). The original SS cone spectral sensitivities have been scaled to unity maximum, but to express them in luminance units instead, we applied the scaling factors that Stockman and Sharpe[7] give.

The choice of units for the cone excitations, particularly for S, has a large influence on the constant-luminance cone-excitation chromaticity diagram.[9,10] For equal-energy white, L values in luminance-based units are close to 70% of total luminance, and M values are close to 30%; this locates the white point near $l = L/(L+M) = 0.7$. In recent work, units for S are most often chosen to make the S value numerically equal to the stimulus luminance in the case of white.[11] Then for white $S = L+M$, or $s = S/(L+M) = 1$.

Here we follow current practice by adopting units such that for an equal-energy white, $s = 1$. For this light, using the SMJ2 and SMJ10 sensitivities results in $l = 0.7$. For SP and SS this chromaticity is given by $l = 0.666$ and $l = 0.647$, respectively.

3. AVERAGE PHOSPHORS

The spectral power distributions of the typical phosphors that we derived by averaging across the set of 15 color monitors that we measured are shown in Fig. 1. The mean CIE 1931 chromaticity coordinates (x, y) of these average phosphor spectra (equal to the mean of the chromaticity coordinates of the measured phosphors) are given in Table 1. These values fall within the range typical of JEDEC P22 phosphors.

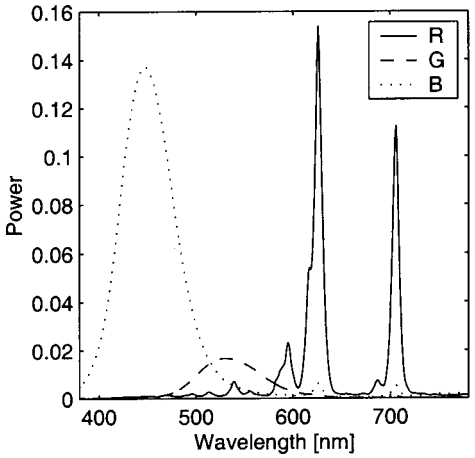

Fig. 1. Spectral power distributions of the typical red, green, and blue phosphors. Before averaging, the individual spectra were scaled for equality in luminance among all phosphors. This leads to the differences in peak power of the three typical phosphors.

Table 1. Mean and Standard Deviation of the CIE 1931 Chromaticity Coordinates for the Sampled Monitors

Chromaticity Coordinate	Phosphor Type		
	Red	Green	Blue
x	0.620 (±0.009)	0.288 (±0.011)	0.153 (±0.004)
y	0.342 (±0.006)	0.597 (±0.008)	0.070 (±0.004)

4. LINEAR TRANSFORMATIONS VALID FOR THE AVERAGE PHOSPHORS

A. Numerical Values for *LMS_XYZ*: Cone Excitations from CIE 1931 Tristimulus Values and Vice Versa

With units for L, M, and S defined as above, the following numerical values for LMS_XYZ were obtained (for formulas see Appendix A) assuming in turn the cone sensitivities specified as (a)–(d) in Section 1.

(a) SP cone excitations from 1931 (X, Y, Z):

$$\begin{pmatrix} L \\ M \\ S \end{pmatrix} = \begin{bmatrix} 0.15282 & 0.54383 & -0.02795 \\ -0.15254 & 0.45524 & 0.03355 \\ -0.00045 & 0.00145 & 0.95449 \end{bmatrix} * \begin{pmatrix} X \\ Y \\ Z \end{pmatrix}. \tag{2}$$

These coefficients are not very different from the defining transformation (cf. p. 615 of Ref. 4) based on the Judd-modified tristimulus values (except for a difference in scaling factor for S). But their adoption eliminates (for our typical phosphors) the errors that arise when the defining transformation is incorrectly applied to the CIE tristimulus values, which as noted above can be as large as 9% in the cone excitation. The error that arises from adopting Eq. (2) with other phosphor spectra is evaluated below, where it proves to be extremely small.

(b) SMJ2 cone excitations for a 2-deg field from 1931 (X, Y, Z):

$$\begin{pmatrix} L \\ M \\ S \end{pmatrix} = \begin{bmatrix} 0.18772 & 0.60445 & -0.02517 \\ -0.14014 & 0.43056 & 0.03773 \\ 0.02017 & -0.04189 & 1.08472 \end{bmatrix} * \begin{pmatrix} X \\ Y \\ Z \end{pmatrix}. \tag{3}$$

(c) SMJ10 cone excitations for a 10-degree field from 1931 (X, Y, Z):

$$\begin{pmatrix} L \\ M \\ S \end{pmatrix} = \begin{bmatrix} 0.14460 & 0.62421 & -0.00429 \\ -0.14506 & 0.42265 & 0.05084 \\ 0.03105 & -0.06416 & 1.10923 \end{bmatrix} * \begin{pmatrix} X \\ Y \\ Z \end{pmatrix}. \tag{4}$$

(d) SS cone excitations for a 2 degree field from 1931 (X, Y, Z):

$$\begin{pmatrix} L \\ M \\ S \end{pmatrix} = \begin{bmatrix} 0.17156 & 0.52901 & -0.02199 \\ -0.15955 & 0.48553 & 0.04298 \\ 0.01916 & -0.03989 & 1.03993 \end{bmatrix} * \begin{pmatrix} X \\ Y \\ Z \end{pmatrix}. \tag{5}$$

The inverse of the above LMS_XYZ transformation gives X, Y, and Z values for given L, M, and S values and can also be very useful for specifying an (X, Y, Z) triplet that will generate a desired (L, M, S) triplet of cone excitations.

For the SMJ2 cone sensitivities, as an example, the inverse transform is

$$\begin{pmatrix} X \\ Y \\ Z \end{pmatrix} = \begin{bmatrix} 2.59795 & -3.62903 & 0.18651 \\ 0.84694 & 1.13166 & -0.01971 \\ -0.01560 & 0.11119 & 0.91767 \end{bmatrix} * \begin{pmatrix} L \\ M \\ S \end{pmatrix}. \tag{6}$$

With the values of LMS_XYZ at hand, one can also determine the transformation from 1931 CIE (x, y) chromaticity coordinates to the (l, s) chromaticity coordinates of MacLeod and Boynton[10] (see Appendix A).

For the SMJ2 cone sensitivities, as an example, the transform is

$$l = \frac{0.21289x + 0.62962y - 0.02517}{0.03502x + 1.0224y + 0.01256},$$

$$s = \frac{-1.06455x - 1.12661y + 1.08472}{0.03502x + 1.0224y + 0.01256}. \tag{7}$$

B. Cone Excitations from Phosphor Luminances Alone

As an alternative to LMS_XYZ, the adoption of a direct and fixed transform from (R, G, B) to (L, M, S) might be considered for a crude estimate of cone excitations from a CRT stimulus in situations where no trusted colorimetric measures at all, but only photometric measures on the red, green, and blue CRT guns, are available. For the SMJ2 cone sensitivities, and for our typical phosphor set, with (R, G, B) expressed in units of CIE 1931 luminance, this is

$$\begin{pmatrix} L \\ M \\ S \end{pmatrix} = \begin{bmatrix} 0.9417 & 0.6901 & 0.7359 \\ 0.1809 & 0.3703 & 0.5432 \\ 0.1138 & 0.1764 & 12.0757 \end{bmatrix} * \begin{pmatrix} R \\ G \\ B \end{pmatrix}. \tag{8}$$

This transform will be in error to the extent that the CRT phosphor spectra are different from the typical ones;

here there is no accommodation of variation among phosphors, such as there is when LMS_XYZ is used in conjunction with colorimetric measures of the CRT stimuli. This naturally leads to much greater error than using colorimetric data, as demonstrated in Subsection 5.C.

C. Improved (X, Y, Z) Values from CIE 1931 Values

Although for CRT stimuli both the cone sensitivities and the tristimulus values can be completely arbitrary without disrupting the linear relationship between tristimulus values and cone excitations, cone sensitivities are often defined as linear combinations of previously defined color matching functions. The SP sensitivities are a linear combination of the Judd-modified CIE color matching functions, and the SMJ10 sensitivities are very close to a linear combination of the 1964 10-deg CIE color matching functions, differing only in the behavior of the S sensitivity at very long wavelengths where it is very low. In such cases, the transformation from CIE 1931 (X, Y, Z) to (L, M, S) can be reduced to two steps: a transformation from (X, Y, Z) to the preferred color matching functions that form the basis for defining L, M and S, followed by the recommended transformation from this new basis to the cone excitations themselves.

The cone excitations then depend linearly on the preferred modified color matching functions, denoted by (X', Y', Z'), as well as on the original 1931 color matching functions:

$$\begin{aligned} LMS &= LMS\_XYZ * XYZ = LMS\_X'Y'Z' * X'Y'Z' \\ &= LMS\_X'Y'Z' * (X'Y'Z'\_XYZ * XYZ). \end{aligned} \tag{9}$$

Here the transformation matrix LMS_XYZ is the product of $LMS\_X'Y'Z'$, the original matrix of coefficients defining the cone sensitivities in terms of the preferred basis (X', Y', Z'), and $X'Y'Z'\_XYZ$, the matrix defining (X', Y', Z') in terms of (X, Y, Z) for the CRT phosphors.

Geometrically, the two sets of tristimulus values provide alternative three-dimensional coordinate systems for representing color and luminance of any stimulus. The coordinate transformation $X'Y'Z'\_XYZ$ maps stimulus coordinates—including those of the R, G, and B phosphors themselves—in the original (X, Y, Z) system into their coordinates in the modified (X', Y', Z') system. It is obtained by a matrix division of the two 3×3 matrices whose rows are the tristimulus values of the R, G, and B phosphors in the two systems. Values for $X'Y'Z'\_XYZ$ to transform 1931 tristimulus values to Judd's (X_J, Y_J, Z_J) (for CRT stimuli only—no linear transformation between these two is even approximately valid in general) are

$$\begin{pmatrix} X_J \\ Y_J \\ Z_J \end{pmatrix} = \begin{bmatrix} 0.98409 & 0.00765 & -0.00140 \\ 0.00046 & 0.99902 & 0.00569 \\ 0.00003 & 0.00052 & 0.93581 \end{bmatrix} * \begin{pmatrix} X \\ Y \\ Z \end{pmatrix}. \tag{10}$$

The further modification of Judd's values by Vos[3] modifies these values only very slightly:

$$\begin{pmatrix} X_V \\ Y_V \\ Z_V \end{pmatrix} = \begin{bmatrix} 0.98398 & 0.00799 & -0.00215 \\ 0.00029 & 0.99911 & 0.00560 \\ -0.00044 & 0.00141 & 0.93294 \end{bmatrix} * \begin{pmatrix} X \\ Y \\ Z \end{pmatrix}. \tag{11}$$

Similarly, the transformation to CIE 1964 large-field values (X_{10}, Y_{10}, Z_{10}) from 1931 tristimulus values for CRT phosphors is

$$\begin{pmatrix} X_{10} \\ Y_{10} \\ Z_{10} \end{pmatrix} = \begin{bmatrix} 0.97008 & 0.09864 & 0.01738 \\ -0.00046 & 1.04684 & 0.04655 \\ 0.02256 & -0.04707 & 1.10164 \end{bmatrix} * \begin{pmatrix} X \\ Y \\ Z \end{pmatrix}. \tag{12}$$

5. ERRORS ENSUING FROM USE OF THE ABOVE TRANSFORMATIONS WITH UNKNOWN PHOSPHORS

A. Errors in Cone Excitations

We now evaluate the errors that arise when the transformations of Subsection 4.A are used to determine cone excitations for individual guns of a CRT from the tristimulus values of the CIE 1931 observer. Here we adopt the SMJ2 cone sensitivities, but the results given are practically identical for other plausible small field cone sensitivities.

CRT stimuli are typically intense enough that detectability of stimulus differences is approximately consistent with Weber's law: Detection depends on the contrast, or fractional change, in each cone's excitation rather than on the absolute difference. Hence the error incurred by the use of the CIE 1931 observer to evaluate L, M, or S for a particular gun of a particular CRT is appropriately represented by expressing it as a fraction of the mean of the cone excitation in question over the set of monitors. To accommodate all the monitor values in a single statistic, we give the root mean square (rms) of these errors for all individual monitors, expressed as a fraction of the average value [Table 2, Fig. 2(a)].

These errors never reach 1 percent and are generally several times smaller than the errors resulting from the application of the original equations of Smith and Pokorny to the 1931 XYZ functions (see Section 1). Moreover, the worst-case error (the largest deviations from the typical value for any individual monitor, expressed as a fraction of the typical value) for each cell in the same format, while naturally larger, exceeds 1% only in the case of the S cones and remains below or comparable with the differential thresholds classically associated with each cone type. The worst-case errors are shown in Table 3.

The given transformations from the CIE 1931 tristimulus values therefore represent with visually negligible error the contrasts formed for each cone type at a boundary between different phosphors.

B. Errors in Chromaticity and Luminance

It is also important to consider the magnitude of the errors in a different format, (l, s, Y), where Y is luminance and l and s are chromaticity coordinates as defined above (see Subsection 2.B). This coordinate system is helpful because particularly small differences in L and M cone excitation can be detected when these are directed along the

isoluminant chromatic l axis.[13] Leonova and MacLeod (see MacLeod[14]), for example, found that differences in l as small as 0.001 could be detected reliably under some conditions in CRT displays. Fortunately the largest error in l resulting from the application of Eq. (3) for any of our monitor phosphors was even less than this, at 0.06% for the blue gun of one monitor. This supports the claim that appropriate transformations of the 1931 CIE observer's tristimulus values, as specified in Subsection 4.A, give rise to negligible error in the absolute specification of CRT stimuli. The rms errors in s and in Y are also very small, not exceeding 1% and 0.25%, respectively (see Table 4).

It is notable that the errors in absolute color specification resulting from the use of the standard observer in combination with our corrective transformation equations would be barely detectable even if stimuli presented on different monitors were directly compared.

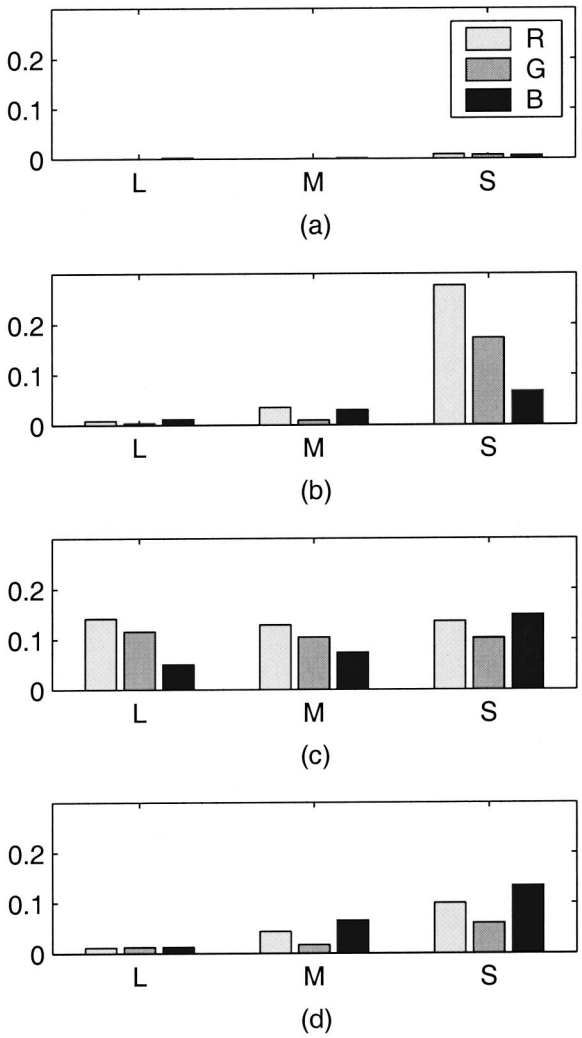

Fig. 2. Errors for L, M, and S cone excitation measures for the R, G, and B phosphors resulting from the following sources: (a) rms errors among monitors resulting from use of the 1931 CIE color matching functions, when the transformation derived for the average monitor [Eq. (3)] is applied to all individual monitors; (b) rms errors among monitors resulting from using R, G, B phosphor intensity measures with the transformation specified in Eq. (8); (c) rms errors among observers resulting from all sources of individual variation for the typical phosphors; (d) rms errors among monitors resulting from differences between measurement devices (based on data provided by Shepherd[12]). Note that the scales of all subplots are equal.

Table 2. Rms Errors Resulting from Use of the 1931 CIE Color Matching Functions with the Transformation Specified in Eq. (3)[a]

Cone Type	Phosphor Type		
	Red	Green	Blue
L	0.0004	0.0010	0.0028
M	0.0005	0.0011	0.0021
S	0.0089	0.0078	0.0064

[a] Each value is the rms of the errors in the estimated L, M, or S cone excitations for any individual monitor, expressed as a fraction of the average excitation for all monitors.

Table 3. Maximum Errors Resulting from Use of the 1931 CIE Color Matching Functions with the Transformation Specified in Eq. (3)[a]

Cone Type	Phosphor Type		
	Red	Green	Blue
L	0.0006	0.0026	0.0053
M	0.0009	0.0026	0.0039
S	0.0263	0.0147	0.0126

[a] Each value is the maximum error in the estimated L, M, or S cone excitations for any individual monitor, expressed as a fraction of the average excitation for all monitors.

Table 4. Rms Errors in Chromaticity and Luminance Resulting from Use of the 1931 CIE Color Matching Functions with the Transformation Specified in Eq. (3)[a]

Colorimetric Measure	Phosphor Type		
	Red	Green	Blue
l	0.00004	0.00002	0.00031
s	0.00909	0.00882	0.00755
Y	0.00039	0.00106	0.00250

[a] Each value is the rms of the errors in the estimated l, s, or Y for any individual monitor, expressed as a fraction of the average value of l, s, or Y for all monitors.

C. Errors in Cone Excitations Based on Phosphor Luminances Alone

Perhaps not surprisingly, the errors introduced by applying the fixed LMS_RGB transformation [Eq. (8)]—effectively assuming, without the use of any colorimetric measurements on the individual monitors, that each monitor has typical phosphor spectra—are approximately 20 times larger than the errors presented in Subsection 5.A (Table 2 and Fig. 2(a)]. These errors are shown in Fig. 2(b).

6. MAGNITUDE OF DIFFERENCES AMONG CANDIDATE CONE SENSITIVITIES

In the estimation of cone excitations (or luminance) for CRT stimuli, the errors arising from the use of the CIE

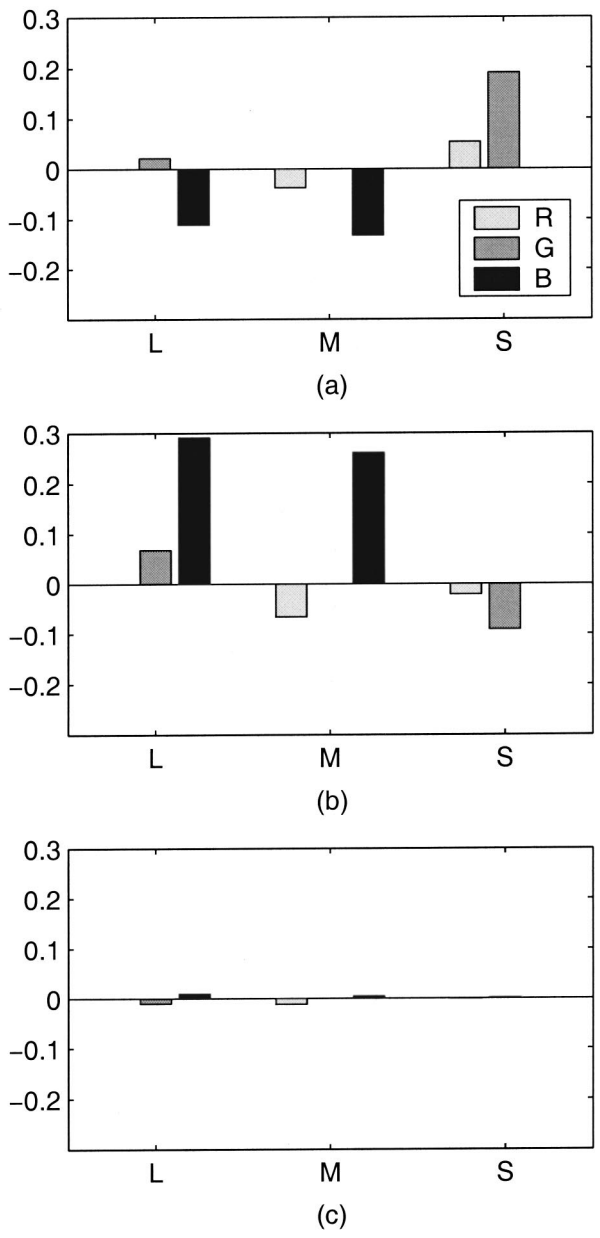

Fig. 3. Differences between candidate cone sensitivities. (a) SP, (b) SMJ10, (c) SS. For each of these three candidate cone sensitivity sets, the fractional deviations of the L, M, and S cone excitation measures for the R, G, and B phosphors from the corresponding SMJ2 values are calculated. A value of +0.1 corresponds to an increase of 10% in cone excitation relative to the SMJ2 value. The effects of different choices of units for cone excitations are eliminated by scaling the estimated L, M, and S values to equality for the red, green, and blue phosphors, respectively.

1931 observer, appropriately handled as in Subsection 4.A, are therefore practically negligible. Two other noninstrumental sources of error can be identified: Observer variation will be investigated in Section 7, and differences in cone excitations entailed by different choices among candidate cone sensitivities are considered now. We compare the candidate cone sensitivities (a)–(d) above, using SMJ2 as a common reference for the others. We eliminated the effects of different choices of units for cone excitations by scaling the estimated L, M, and S values to equality for the red, green and blue phosphors respectively. By comparison with SMJ2, the SP sensitivities underestimate the L and M excitations by the blue phosphor by approximately 14% and overestimate the S cone excitation by the green phosphor by 19% [Fig. 3(a)]. These differences far exceed the inconsequential errors resulting from use of the transformations of Subsection 4.A and even exceed the errors, cited in Section 1, that result from naïve identification of the 1931 tristimulus values with the Judd ones.

The SMJ10 degree functions naturally show even greater differences from the small-field SMJ2 sensitivities, especially in the effects of the blue phosphor on the L and M cones, which exceed the small field values by approximately 26% owing to the reduced average density of macular pigment that prevails in the larger field [Fig. 3(b)].

The SS excitations, however, are very close to the SMJ2 values, in spite of being based on a slightly different set of color matching functions [Fig. 3(c)]. The rms difference between these two sets of cone excitations for CRT phosphors is approximately 1%. The greater differences implicit in transformations (3) and (5) above originate from a different choice of luminosity function, which occasioned a slightly different choice of units for L, M, and S: The SMJ2 function is based on the CIE 1964 $\bar{y}_{10}(\lambda)$, corrected for small field conditions,[1] whereas the SS sensitivity is based on a more recent set of measurements.[7]

Figure 4 shows the locus of spectral lights as well as the equal-energy white point in the (l, s) plane for the four candidate cone sensitivities. The plots emphasize differences in the short wavelengths, but these are in practice significant only for monochromatic lights. The Stockman and Sharpe[7] s value has a local minimum at 395 nm, where a concavity originating in the Stiles and Burch color matching functions was removed in defining the 1964 CIE 10-deg observer. What is more important, the l coordinates of the spectral colors and of the white point in this representation depend considerably on the choice of the coefficients with which the L and M excita-

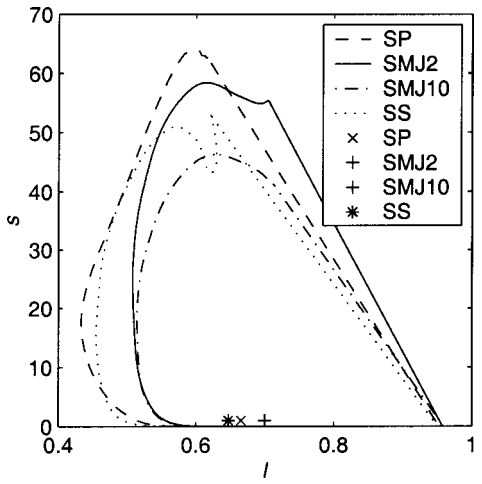

Fig. 4. Locus of spectral lights (curves) and equal-energy white point (symbol markers) of each of the four candidate cone sensitivities introduced in Section 1. The l axis is the luminance-normalized L-cone excitation, and the s axis is the luminance-normalized S-cone excitation.

tions are combined to determine luminance. The cone sensitivities (a)–(d) are associated with four different luminosity functions: respectively, $\bar{y}_J(\lambda)$, $\bar{y}_2(\lambda)$ obtained with a small-field correction from $\bar{y}_{10}(\lambda)$, $\bar{y}_{10}(\lambda)$, and new luminosity measurements by Stockman and Sharpe.[7] Figure 4 thus illustrates an unfortunate feature of the constant-luminance chromaticity diagram: The diagram introduces photometric issues into the colorimetric specification of stimuli by adopting coordinates that depend on the L and M luminance coefficients.

7. OTHER SOURCES OF UNCERTAINTY: OBSERVER VARIATION

No set of cone sensitivities can be correct for all observers, since different observers have slightly different sensitivities. This individual variation limits the accuracy with which color can be specified for particular observers, even if accurate radiometric data are available. Useful evidence as to the nature and extent of individual variation was obtained by Webster and MacLeod[15] in a factor analysis of the color matching functions of Stiles and Burch's[5] 49 observers. They found six identifiable and independent factors contributing to the individual variation. First, the optical density of the macular pigment that screens the foveal cones varied with a standard deviation of 0.15 at 460 nm in a 2-deg field, approximately 40% of its mean density across observers. Second, the optical density of the lens that screens all receptors varied with a standard deviation of 0.18 at 400 nm, approximately 19% of its mean density across observers. In addition, the wavelengths of peak absorption of each of the three light-absorbing cone pigments varied independently, presumably as a result of polymorphism in the genes encoding the pigments,[16] with standard deviations of 1.5 nm (L), 0.9 nm (M), and 0.8 nm (S). Finally, the density of the visual pigments varied together, perhaps as a result of individual variation in outer segment length, with a standard deviation of 0.045. The effects of these variations on cone excitation by CRT phosphors are straightforward to evaluate (for two previous studies on this topic see Refs. 17 and 18). We consider, as an index of the individual variation contributed by any one physiological factor, the fractional rms variation in cone excitation due to that factor. By fractional rms variation we mean the rms observer variation (or standard deviation) divided by the mean across observers. The variations are small enough that effects are roughly proportional to the responsible differences in the underlying factors, so the rms variation in cone excitation due to each causative factor separately is equal (disregarding the sign) to the change associated with a change of 1 standard deviation in the underlying cause of variation. Accordingly, we next evaluate, as an index of observer variation, these standard-deviation changes for each factor and for each cone excitation in turn. Results are presented as fractional changes, that is, the change in cone excitation when the corresponding observer variable increases by 1 standard deviation, expressed as a fraction of the mean cone excitation across observers. The fractional change is thus equal to the percent change divided by 100.

A. Cone Pigment Polymorphism

Assuming that individual variation in visual-pigment-absorption spectra displaces them on a log-wavelength scale,[15] with the standard deviations given above at the wavelengths of peak absorption, we find the following fractional changes in each cone's excitation when its wavelength of peak absorption λ_{max} is incremented by 1 standard deviation (Table 5).

The absolute values from Table 5 give the rms observer variation due to cone pigment polymorphism. These effects, maximally approximately 4%, are much larger than the errors resulting from the adoption of the standard observer with the optimized transformations of Subsection 4.A, though still not as large as the differences between the SP and SMJ2 sensitivities.

When an observer's individual cone sensitivities can be estimated, deviations in an observer's λ_{max} from the population average affect the cone excitations received from CRT stimuli as shown in Table 6. The tabled numbers are the fractional increments in excitation per nm of displacement from the population average λ_{max}.

The serine/alanine polymorphism at site 180 of the L pigment gene, for example, which has been reported to give rise to bimodal variation in the L sensitivity among males, with a separation of about 2.5 nm between the modes,[16,19] generates a 2.5×0.028, or 7%, change in sensitivity for the red phosphor.

Table 6 has another use: when cone excitation estimates are based on spectroradiometric data, Table 6 gives the error introduced per 1 nm error in the wavelength calibration of the instrument.

Table 5. Rms Errors Resulting from Observer Variation in Visual Pigment Absorption Spectra for Our Average Monitor Phosphors[a]

Cone Type	Phosphor Type		
	Red	Green	Blue
L	0.0417	−0.0008	−0.0158
M	0.0363	0.0062	−0.0100
S	0.0293	0.0552	0.0200

[a] Each value is the fractional change in the estimated L, M, or S cone excitations when the corresponding wavelength of peak absorption, λ_{max} is increased by 1 standard deviation; if negative signs are disregarded, the values are the rms fractional variations across observers in cone excitation that result from observer variation in λ_{max}.

Table 6. Changes in Cone Excitations Resulting from Displacements of Visual Pigment Absorption Spectra[a]

Cone Type	Phosphor Type		
	Red	Green	Blue
L	0.0278	−0.0005	−0.0105
M	0.0404	0.0069	−0.0111
S	0.0366	0.0690	0.0250

[a] Each value is the fractional increment in the estimated L, M, or S cone excitations when the corresponding peak absorption is increased by 1 nm.

Table 7. Rms Errors Resulting from Individual Variation in Macular Pigmentation[a]

Cone Type	Phosphor Type		
	Red	Green	Blue
L	0.0037	0.0272	0.1764
M	0.0152	0.0375	0.2080
S	0.2715	0.2341	0.2829

[a] Each value is the rms of the fractional changes in the estimated L, M, or S cone excitations due to observer variation in macular pigmentation

Table 8. Rms Errors Resulting from Individual Variation in Lens Pigmentation[a]

Cone Type	Phosphor Type		
	Red	Green	Blue
L	0.0067	0.0209	0.0408
M	0.0122	0.0231	0.0447
S	0.0718	0.0499	0.0787

[a] Each value is the rms of the fractional changes in the estimated L, M, or S cone excitations due to observer variation in lens pigmentation.

Table 9. Rms Errors Resulting from Individual Variation in Visual Pigment Density[a]

Cone Type	Phosphor Type		
	Red	Green	Blue
L	0.0797	0.0624	0.0780
M	0.0831	0.0643	0.0751
S	0.0755	0.0874	0.0710

[a] Each value is the rms of the fractional changes in the estimated L, M, or S cone excitations due to observer variation in visual pigment density.

B. Macular and Lens Pigment Variation

A reduction by 1 standard deviation (0.15 in peak density) generates the fractional increments in the cone excitations shown in Table 7.

Macular pigmentation strongly influences the S cone excitation (bottom row) and all responses to the blue phosphor (final column). In these cases the effects of a single standard deviation of observer variation are as large as the largest differences between candidate cone sensitivities. The effects on the L and M cone contrasts evoked by the red and green phosphors are much smaller yet are still comparable with the effects of cone polymorphism—a change of a few per cent in the relative effectiveness of the red and green phosphors.

Variation in the optical density of the lens is less important. A reduction of 1 standard deviation (10% in density) yields the fractional increments in cone excitation shown in Table 8.

C. Visual Pigment Density

An increment in visual pigment density by 1 standard deviation (by 0.045 in density, or 10% if the mean density is taken as 0.45) generates the fractional increments in the cone excitations shown in Table 9.

Variation in visual pigment density increases or decreases all cone excitations together, behaving mainly like variation in intensity independent of the stimulus, so its effects on color are actually much less than the numbers in Table 9 might suggest. Variation in effective stimulus intensity across observers can be normalized out of Table 9 by scaling all cone excitations, for all phosphors, by the same observer-dependent factor, chosen to make the mean of all three cone excitations the same for all observers. When this is done, the rms effects of pigment density variation are much reduced, as shown in Table 10.

This normalization also reduces the effects of the other cited sources of individual variation, but only slightly.

D. Combined Effects of All Sources of Observer Variation: Cone Excitations

All the independent sources of individual variation can now be combined by (1) discarding the variation in effective stimulus intensity due to each factor, as was done for visual pigment density in Table 10, by normalizing out variation common to all phosphors and all cones; (2) adding the resulting variances due to each factor; and (3) taking the square root of the result. The resulting fractional standard deviations for variation between observers [Table 11, Fig. 2(c)] are dominated by the macular pigment factor. They reveal that observer differences are comparable in magnitude with the largest differences between competing cone-sensitivity estimates.

E. Combined Effects of All Sources of Observer Variation: Cone Contrasts

The numbers given in Table 11 express the combined effect of all factors on the excitation of individual cones by individual phosphors, but because the variations represented by different cells are correlated, they do not directly indicate the extent of individual variation in, for example, the cone contrasts formed between display regions lit by different phosphors. To evaluate variation in cone contrast, we can normalize to reject changes in cone exci-

Table 10. Rms Errors Resulting from Individual Variation in Visual Pigment Density[a]

Cone Type	Phosphor Type		
	Red	Green	Blue
L	0.0045	−0.0127	0.0028
M	0.0079	−0.0109	0.0000
S	0.0003	0.0122	−0.0042

[a] This differs from Table 9 because changes common to all cone excitations and all phosphors are discounted here by normalization with the same scaling factor.

Table 11. Rms Errors Resulting from All Sources of Individual Variation

Cone Type	Phosphor Type		
	Red	Green	Blue
L	0.1417	0.1161	0.0501
M	0.1287	0.1046	0.0741
S	0.1364	0.1029	0.1489

Table 12. Rms Errors Resulting from all Sources of Individual Variation[a]

Cone Type	Phosphor Type		
	Red	Green	Blue
L	0	0.0491	0.1839
M	0.0385	0	0.1665
S	0.0149	0.0624	0

[a] For each cone type, changes common to all phosphors are discounted by normalization. Thus the values indicate individual variation of cone contrast instead of variation of absolute cone excitation.

tation by factors that are different for different cones but identical for all phosphors. These effects have no influence on color matches or on the cone contrasts created by color displays. (They can, however, influence some aspects of color vision such as the subjectively achromatic point or the unique yellow wavelength, so it could be misleading, in those contexts, to normalize them out.)

If overall changes in L, M, S for each factor are normalized so as to make the diagonals zero before combining them, macular pigment remains the predominant factor, but the standard deviations for L and M cone excitations by the red and green phosphors drop to a few per cent. The matrix then obtained by combining all factors indicates the total individual variation in relative effectiveness of the three phosphors for each cone type in turn (see Table 12).

These numbers make it possible to compute the uncertainty in cone contrast introduced by observer variation with color CRT displays. For instance, if a bipartite field with the red phosphor on one side and the green phosphor on the other is arranged to provide zero contrast for the L cones of the average observer, the rms Michelson contrast formed for the L cones of all observers will be $(1.0491 - 1)/(1.0491+1)$, or 2.4%. This illustrates the difficulty of arranging a "silent exchange" CRT stimulus without knowledge of each individual observer's characteristics. The nominally zero cone contrast will be detectable, albeit barely, for a large proportion of the population. Much greater uncertainty arises if the blue phosphor is used instead of the green, since the blue phosphor is strongly absorbed in macular pigment. Local variation across the retina then becomes important also, as noted in Section 6.

F. Luminance

In addition to cone contrasts, luminance ratios are critical for the perception of color displays. Luminance is well described by the appropriately weighted sum of the L and M cone excitations. Of interest to CRT users are the relative luminances of the phosphors, for instance of the red and blue phosphors relative to the green one. Strict adherence to the terminology proposed by the CIE (where luminance is defined without reference to human observers), actually precludes observer variation in luminance; here, following Kaiser,[20] we will therefore adopt the term "sensation luminance" to refer to luminance as assessed for a particular observer by using the accepted photometrically additive procedures, principally flicker photometry.

Individual variation in relative sensation luminance will be partly traceable to individual variation in the physiological colorimetric factors considered above; additional variation may be generated by observer variation in the relative weighting of the L and M cones for sensation luminance. We next give quantitative estimates for the contribution of the colorimetric factors to individual variation in CRT isoluminance settings. Observer variance in excess of this amount (but *only* the excess variance) will be attributable to observer variation in the relative weights for sensation luminance of the L and M cones. The way that variation in colorimetric factors might inflate estimated variation in the L/M cone weighting for sensation luminance has been previously documented, but only for monochromatic stimuli and without consideration of the actual magnitudes of individual variation.[21]

Each of the forms of observer variation considered above contributes some variance in relative sensation luminance of the phosphors. Relative sensation luminance is reflected in the isoluminant setting, where the intensity of one phosphor is varied to achieve equality in sensation luminance with a fixed intensity of a reference stimulus (typically with a flicker-photometric or minimum-motion criterion). The relative sensation luminance is the reciprocal of the isoluminant intensity. We consider, as an index of the individual variation contributed by any one physiological factor, the fractional rms variation in relative sensation luminance due to that factor. As before, by fractional rms variation we mean the rms observer variation (or standard deviation) divided by the mean across observers.

Table 13 shows the fractional change in sensation luminance of the red and blue phosphors (relative to the green phosphor) caused by an increase of 1 standard deviation in the listed physiological factor. The maximum effect for the red phosphor is less than 4%, caused by an increase of 1 standard deviation (1.5 nm) in L cone λ_{max}. For the blue phosphor, an increase in macular pigment density by 1 standard deviation causes an 18% reduction in relative sensation luminance.

Table 13. Fractional Changes in Sensation Luminance of the Red and Blue Phosphor Relative to That of the Green Phosphor Resulting from Different Sources of Individual Variation[a]

Source of Variation	Phosphor Type	
	Red	Blue
L cone λ_{max}	0.0362	−0.0085
M cone λ_{max}	0.0037	−0.0064
Macular pigment density	0.0249	−0.1751
Lens density	0.0139	−0.0211
Visual pigment density	0.0175	0.0133
Total rms variation	0.0494	0.1772

[a] Each value is the fractional increment in sensation luminance when the corresponding physiological factor is increased by 1 standard deviation.

In the final row of Table 13, all the independent sources of individual variation have been combined by adding the variances due to them and taking the positive square root of the result. This is the standard deviation for variation between observers, in sensation luminance relative to the green phosphor, that would be expected from the variation in spectral filtering factors alone, without any variation in the weighting of the L and M cone contributions to sensation luminance. The variance attributable to the latter can be estimated by subtracting the square of the total rms variation given above from the variance experimentally observed. For example, in recent data of Gunther and Dobkins,[22] the true variation in the sensation luminance of the red phosphor relative to the green for their 41 observers was 6.7%. Subtracting from total true variance (the square of this) the calculated variance due to the spectral filtering factors (which is the square of 0.0494 from Table 13), we obtain the standard deviation that would in theory have resulted if those factors were constant (and only cone weighting varied across observers):

$$\sqrt{0.067^2 - 0.0494^2} = 0.0453.$$

This variability attributable to cone weighting alone is here approximately 2/3 of true total standard deviation. Allowance for this would cause the estimated range in cone weights among observers to be constricted by approximately the same 2/3 factor.

The correlation between estimated and true cone weights would be $\sqrt{2/3}$, or just over 0.8. Thus, we conclude that observer variation in the visual pigments and screening pigments is not enough to disrupt very seriously the relation between a subject's L and M cone weights and the subject's isoluminance setting for the red and green phosphors. In a luminance match between the blue and green phosphors, however, the contribution of the colorimetric factors, particularly macular pigment, will be predominant. Individual variation in L and M cone weights alone generates an rms observer variation of less than 3% in this setting, whereas the variation generated by colorimetric factors alone is 18% (Table 13).

8. OTHER SOURCES OF UNCERTAINTY: COLORIMETRIC DEVICES

Simple tri-filter colorimeters are reliable but they often lack accuracy.[4,12] These devices measure the 1931 CIE X, Y, Z tri-stimulus values by three sensors whose light sensitivities approximate the CIE $\bar{x}(\lambda)$, $\bar{y}(\lambda)$, and $\bar{z}(\lambda)$ functions. For this purpose, combinations of colored glass filters are used such that the resulting spectral transmittance function fits the respective CIE color matching function. But an accurate fit is hard to achieve, especially for the double-peaked $\bar{x}(\lambda)$ function.[4]

One such tri-filter colorimeter, a Minolta CS-100, was tested by Shepherd.[12] To evaluate the accuracy of this device, she compared its readings for the phosphor lights from two CRTs with the CIE values calculated from the spectroradiometric measurements by a Photo Research Spectra Scan. To make the deviations between these two devices comparable with the errors that we reported above, we transformed the data underlying Fig. 1 of Ref. 12 into fractional errors in L, M, S cone excitations. The rms fractional error of the two CRTs (deviations between the two devices being very similar for the two CRTs) is as large as 21% for the L cone excitation from the blue phosphor.

Table 14. Rms Errors Resulting from Measurement-Device Variation[a]

Cone Type	Phosphor Type		
	Red	Green	Blue
L	0.0116	0.0129	0.0128
M	0.0432	0.0171	0.0651
S	0.0995	0.0601	0.1344

[a] Based on the differences between a trifilter colorimeter and a spectroradiometer measured by Shepherd.[12] Differences in luminance have been corrected for (see text). Each value is the rms of the differences of the two devices in L, M, or S cone excitations for two monitors, expressed as a fraction of the average spectroradiometrically determined excitation for both monitors.

The luminance readings of the Minolta CS-100 were on average 10% lower than the values determined spectroradiometrically. Since this difference in favor of the Spectra Scan existed consistently for all phosphor measurements of all CRTs and could be caused by calibration differences,[12] we corrected for these luminance differences: We substituted the Minolta CS-100 luminance readings by the spectroradiometrically determined values and used them along with the unchanged Minolta CS-100 chromaticity readings. The resulting differences in cone excitations between these two devices are shown in Table 14 [see also Fig. 2(d)]

These deviations are in agreement with the degree of inaccuracy common for simple tri-filter colorimeters.[4] One major source of the deviation of the Minolta CS-100 readings from the spectroradiometrically determined values is possibly its unsatisfactory approximation of $\bar{x}(\lambda)$: The Minolta CS-100 uses a scaled version of $\bar{z}(\lambda)$ to approximate the short-wavelength peak of $\bar{x}(\lambda)$.[12] More-elaborated colorimeters with filters built on the basis of the Dresler principle[4] are available commercially nowadays. They have substantially improved accuracy, and only such colorimeters should be used in colorimetric measurements of CIE X, Y, Z values that require the accuracy usually required for vision science.

APPENDIX A: FORMULAS

1. Notation

$\bar{x}(\lambda)$, $\bar{y}(\lambda)$, and $\bar{z}(\lambda)$ are the 1931 CIE color matching functions. $\bar{x}_J(\lambda)$, $\bar{y}_J(\lambda)$, and $\bar{z}_J(\lambda)$ are the Judd-modified CIE color matching functions. $L(\lambda)$, $M(\lambda)$, and $S(\lambda)$ are the spectral cone-sensitivity functions (for any of the SP, SMJ2, SMJ10, or the SS candidate set, see Section 1, list (a)–(d) and with units as described in Subsection 2.B). $r_i(\lambda)$, $g_i(\lambda)$, and $b_i(\lambda)$ are the spectral power distributions of the red, green, and blue phosphor of monitor i. All spectral functions were defined in the range from 380 to 780 nm in 1-nm steps. Where necessary this format was derived by using cubic spline interpolation. INV( ) denotes the inverse of a matrix and $(\,)^T$ the transpose.

2. Section 3

The spectral power distributions of the typical red, green, and blue phosphor $\bar{r}(\lambda)$, $\bar{g}(\lambda)$ and $\bar{b}(\lambda)$ were calculated by averaging over all 15 measured monitors:

$$\bar{r}(\lambda) = \sum_i r_i(\lambda), \qquad \bar{g}(\lambda) = \sum_i g_i(\lambda),$$

$$\bar{b}(\lambda) = \sum_i b_i(\lambda) \qquad (\lambda = 380,..,780).$$

3. Subsection 4.A

The LMS_XYZ transformation matrices were calculated as

$$LMS\_XYZ = \begin{pmatrix} L(\lambda) \\ M(\lambda) \\ S(\lambda) \end{pmatrix} * \begin{pmatrix} \bar{r}(\lambda) \\ \bar{g}(\lambda) \\ \bar{b}(\lambda) \end{pmatrix}^{\mathrm{T}} * \mathrm{INV}\left(\begin{pmatrix} \bar{x}(\lambda) \\ \bar{y}(\lambda) \\ \bar{z}(\lambda) \end{pmatrix} * \begin{pmatrix} \bar{r}(\lambda) \\ \bar{g}(\lambda) \\ \bar{b}(\lambda) \end{pmatrix}^{\mathrm{T}} \right).$$

Let

$$\begin{bmatrix} L_X & L_Y & L_Z \\ M_X & M_Y & M_Z \\ S_X & S_Y & S_Z \end{bmatrix}$$

denote the elements of such a LMS_XYZ matrix for one of the four sets of candidate cone sensitivities. Then the corresponding transform from 1931 CIE (x, y) chromaticity coordinates to the (l, s) chromaticity coordinates of MacLeod and Boynton[10] is determined by

$$l = \frac{(L_X - L_Z) * x + (L_Y - L_Z) * y + L_Z}{(L_X + M_X - L_Z - M_Z) * x + (L_Y + M_Y - L_Z - M_Z) * y + L_Z + M_Z},$$

$$s = \frac{(S_X - S_Z) * x + (S_Y - S_Z) * y + S_Z}{(L_X + M_X - L_Z - M_Z) * x + (L_Y + M_Y - L_Z - M_Z) * y + L_Z + M_Z}.$$

4. Subsection 4.B

The LMS_RGB transformation matrices were calculated as

$$LMS\_RGB = \begin{pmatrix} L(\lambda) \\ M(\lambda) \\ S(\lambda) \end{pmatrix} * \begin{pmatrix} \bar{r}(\lambda) \\ \bar{g}(\lambda) \\ \bar{b}(\lambda) \end{pmatrix}^{\mathrm{T}}.$$

5. Subsection 4.C

The $X'Y'Z'\_XYZ$ matrices for the transformation from 1931 CIE to Judd-modified tristimulus values were calculated as

$$X'Y'Z'\_XYZ = \begin{pmatrix} \bar{x}_J(\lambda) \\ \bar{y}_J(\lambda) \\ \bar{z}_J(\lambda) \end{pmatrix} * \begin{pmatrix} \bar{r}(\lambda) \\ \bar{g}(\lambda) \\ \bar{b}(\lambda) \end{pmatrix}^{\mathrm{T}} * \mathrm{INV}\left(\begin{pmatrix} \bar{x}(\lambda) \\ \bar{y}(\lambda) \\ \bar{z}(\lambda) \end{pmatrix} * \begin{pmatrix} \bar{r}(\lambda) \\ \bar{g}(\lambda) \\ \bar{b}(\lambda) \end{pmatrix}^{\mathrm{T}} \right).$$

The matrices for transforming 1931 CIE to Vos-modified and to 1964 CIE tristimulus values were calculated similarly, only differing in that $\bar{x}_J(\lambda)$, $\bar{y}_J(\lambda)$ and $\bar{z}_J(\lambda)$ were replaced by the respective color matching functions.

6. Section 5

Errors in cone excitations ensuing from use of Eq. (3) were calculated as follows: With the phosphor spectra at hand, for each of the 15 monitors a matrix LMS_XYZ_i for transforming 1931 CIE XYZ values to LMS cone excitations can be calculated that is exactly correct for the respective monitor:

$$LMS\_XYZ_i = \begin{pmatrix} L(\lambda) \\ M(\lambda) \\ S(\lambda) \end{pmatrix} * \begin{pmatrix} r_i(\lambda) \\ g_i(\lambda) \\ b_i(\lambda) \end{pmatrix}^{\mathrm{T}} * INV\left(\begin{pmatrix} \bar{x}(\lambda) \\ \bar{y}(\lambda) \\ \bar{z}(\lambda) \end{pmatrix} * \begin{pmatrix} \bar{r}(\lambda) \\ \bar{g}(\lambda) \\ \bar{b}(\lambda) \end{pmatrix}^{\mathrm{T}} \right)$$

$$(i = 1,.., 15).$$

For each of the average phosphor spectra $\bar{r}(\lambda)$, $\bar{g}(\lambda)$ and $\bar{b}(\lambda)$ the estimated LMS values were calculated

(a) by using the transformation LMS_XYZ based on the average phosphors [Eq. (3) of Subsection 4.A step (b)]:

$$\begin{bmatrix} L_{\bar{r}} & L_{\bar{g}} & L_{\bar{b}} \\ M_{\bar{r}} & M_{\bar{g}} & M_{\bar{b}} \\ S_{\bar{r}} & S_{\bar{g}} & S_{\bar{b}} \end{bmatrix} = LMS\_XYZ * \left(\begin{pmatrix} \bar{x}(\lambda) \\ \bar{y}(\lambda) \\ \bar{z}(\lambda) \end{pmatrix} * \begin{pmatrix} \bar{r}(\lambda) \\ \bar{g}(\lambda) \\ \bar{b}(\lambda) \end{pmatrix}^{T} \right);$$

(b) by using the above transformations LMS_XYZ_i based on the phosphors of each individual monitor:

$$\begin{bmatrix} L_{\bar{r}} & L_{\bar{g}} & L_{\bar{b}} \\ M_{\bar{r}} & M_{\bar{g}} & M_{\bar{b}} \\ S_{\bar{r}} & S_{\bar{g}} & S_{\bar{b}} \end{bmatrix}_i = LMS\_XYZ_i * \left(\begin{pmatrix} \bar{x}(\lambda) \\ \bar{y}(\lambda) \\ \bar{z}(\lambda) \end{pmatrix} * \begin{pmatrix} \bar{r}(\lambda) \\ \bar{g}(\lambda) \\ \bar{b}(\lambda) \end{pmatrix}^{T} \right).$$

$$(i = 1,.., 15)$$

Now the rms of the differences of the LMS estimate based on the average phosphor from the exact LMS estimates based on each individual monitor was calculated and normalized by the LMS estimate based on the average phosphor:

$$\text{fractional rms}\begin{bmatrix} L_{\bar{r}} & L_{\bar{g}} & L_{\bar{b}} \\ M_{\bar{r}} & M_{\bar{g}} & M_{\bar{b}} \\ S_{\bar{r}} & S_{\bar{g}} & S_{\bar{b}} \end{bmatrix} = \sqrt{1/15\sum_i \left(\begin{bmatrix} L_{\bar{r}} & L_{\bar{g}} & L_{\bar{b}} \\ M_{\bar{r}} & M_{\bar{g}} & M_{\bar{b}} \\ S_{\bar{r}} & S_{\bar{g}} & S_{\bar{b}} \end{bmatrix} - \begin{bmatrix} L_{\bar{r}} & L_{\bar{g}} & L_{\bar{b}} \\ M_{\bar{r}} & M_{\bar{g}} & M_{\bar{b}} \\ S_{\bar{r}} & S_{\bar{g}} & S_{\bar{b}} \end{bmatrix}_i \right)^2} \Big/ \begin{bmatrix} L_{\bar{r}} & L_{\bar{g}} & L_{\bar{b}} \\ M_{\bar{r}} & M_{\bar{g}} & M_{\bar{b}} \\ S_{\bar{r}} & S_{\bar{g}} & S_{\bar{b}} \end{bmatrix}$$

where $\sqrt{\ }$, $(\,)^2$, and / are scalar element-by-element operations. All other error values presented in Section 5 were calculated accordingly.

APPENDIX B: SUPPLEMENTARY MATERIAL

The phosphor data set of our average phosphors and individual monitors, color matching and cone sensitivity functions as used in our paper, and a MATLAB script to calculate the transform matrices based on the average phosphors (Section 4) and to determine transform matrices that are exact for your own CRT are available at www.psychologie.uni-kiel.de/golz/publications/4/supplement.html.

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