Unemployment Invariance

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Abstract

This paper provides a critique of the "unemployment invariance hypothesis", according to which the behavior of the labor market, by itself, ensures that the long-run unemployment rate is independent of the size of the capital stock, productivity, and the labor force. In the context of an endogenous growth model, we show that the labor market alone need not contain all the equilibrating mechanisms to ensure unemployment invariance; in particular, other markets may perform part of the equilibrating process as well. By implication, policies that raise the growth path of capital or increase the effective working-age population may influence the long-run unemployment rate.

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1 Introduction

Is long-run unemployment affected by policies that raise the capital stock (all other things equal, including the rate of capital accumulation)? Would the long-run unemployment rate be influenced by a rise in the effective working-age population (induced, say, by early retirement measures or constraints on working time), or an increase in productivity (generated, for example, by policies promoting R&D), other things being equal? Questions of this sort have been central to the policy debate concerning unemployment over the past few decades.

The mainstream answer to these questions in the macro labor economics literature has been dominated by the need to explain why unemployment rates in the OECD are trendless in the long run (e.g. over the past century), despite growth in the capital stock, total factor productivity, and the labor force. Productivity growth stimulated labor demand; population growth stimulated labor supply. But these developments have not proceeded at the same rate. For most OECD countries, the rate of productivity growth over the past century has far exceeded the rate of population growth, but nevertheless the unemployment rates have not followed a declining trend over the long run.[1]

The way in which the mainstream literature has captured this phenomenon is through what may be called the "unemployment invariance hypothesis," which asserts that the behavior of the labor market, by itself, ensures that the long-run unemployment rate is independent of the size of the capital stock, total factor productivity and the labor force. This hypothesis is illustrated in Figure 1, which pictures an aggregate labor demand curve (LD, specifying aggregate employment at any given real wage), a wage setting curve (WS, specifying the equilibrium real wage any any given aggregate employment level),[2] and a labor supply curve (LS, specifying the size of the labor force at any given real wage, depicted for simplicity by a vertical line). The intersection of the labour demand curve LD_1 and the wage setting curve WS_1 yields the initial equilibrium employment level (E_1^*) and the equilibrium real wage (w_1^*). The difference between the labour supply (LS) and employment at the equilibrium real wage (w_1^*) is the equilibrium unemployment

[1] Moreover, the major swings in labor force growth, in response to baby booms and troughs, have not been closely related to the major swings in productivity growth. However, there is no evidence of any trends in unemployment related to these developments.

[2] If the labor market clears, the wage setting curve coincides with the labor supply curve. If it does not clear - for efficiency wage, insider-outsider, labor union, or other reasons - then the wage setting curve lies to the left of the labor supply curve, as illustrated in the figure.

level (U^*). Now suppose that the labor demand curve shifts outwards to LD_2, due to capital accumulation or technological advance. Then, according to the unemployment invariance hypothesis, the wage setting and/or labor supply curves must shift in the long run so that the unemployment rate remains unchanged. The standard way of achieving this result is to specify wage setting behavior so as to ensure that the wage setting curve shifts inwards by the same amount as the labor demand curve shifts out - i.e. to WS_2 - so that equilibrium unemployment (and unemployment rate) are the same.

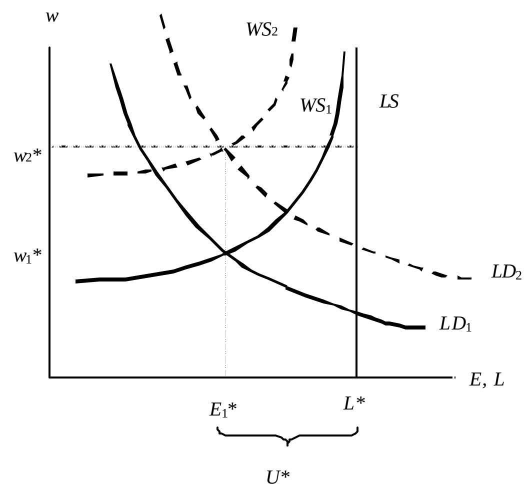

Fig. 1: The Unemployment Invariance Hypothesis

Similarly, if the labor supply curve LS shifts outwards due to population growth, the wage setting curve may shift outward in the long run, once again leaving the unemployment rate unchanged.

The literature contains two influential forms of the unemployment invariance hypothesis. The "strong invariance" hypothesis (e.g. Layard, Nickell and Jackman (1991)) asserts that any exogenous permanent shock in the capital stock, total factor productivity, or the labor supply leads to counterveiling shifts in the labor demand, wage setting, and labor supply curves so as to restore the unemployment rate to its original long-run equilibrium. By implication, policies that stimulate capital accumulation or R&D or policies that reduce the size of the labor force (cœteris paribus) can have no effect on the long-run unemployment rate.

The "weak invariance" hypothesis (e.g. Phelps (1994, ch. 17), Fitoussi, Jestaz, Phelps, and Zoega (2000)) assert that the long-run unemployment rate can be influenced by the capital stock, productivity and the labor force only in trendless combinations. For instance, given that the ratio of capital to labor (in efficiency units) is trendless, then the long-run unemployment rate may depend on this ratio. This weak invariance hypothesis is also supported by Rowthorn (1999). He argues that the capital stock does not affect the long-run unemployment rate in the Layard-Nickell-Jackman model only because this model assumes a Cobb-Douglas production function, so that the elasticity of substitution between labor and capital is unity. If, more realistically, this elasticity is taken to be less than unity, then weak invariance follows.[3]

It is common practice, in empirical modeling of the aggregate labor market, to impose restrictions on the labor demand, wage setting and labor supple curves that make the unemployment invariance hypothesis (in its strong or weak form) hold.[4] These restrictions generally play a crucially important role in conditioning the behavior of these models. The empirical illustration we provide in Section 5 shows that they are rejected by the data. In addition, Phelps (1994) estimates a reduced form unemployment equation which includes capital stock as an explanatory variable.[5] Arestis and Mariscal (2000), using quarterly data for the UK and Germany, establish significant effects of capital stock on unemployment and conclude that investment enhancing policies could have a permanent effect on unemployment in both countries. In short, these restrictions are an instance of theory overriding empirical considerations.

This paper calls the unemployment invariance hypothesis into question. In particular, we argue that the weak and strong forms of this hypothesis, applied to labor market activity, are not necessary to explain the long-run trendlessness of unemployment. The reason is that both forms of the hypothesis imply the long-run unemployment trendlessness is due entirely to equilibrating mechanisms in the labor market. By contrast, we show that the labor market need not, all by itself, ensure such trendlessness. Instead, what is required is merely that all the markets in the economy, interacting with one another, generate such equilibrating mechanisms.

In short, the labor market may be only one of various markets doing the required equilibration to generate trendlessness of unemployment over the long run. The fact that the unemployment invariance restrictions are usually rejected by the data suggests that this is indeed generally the case.

It may be tempting to believe that when all markets in the economy are involved in the equilibration process that ensures long-run unemployment trendlessness, then unemployment invariance restrictions (strong or weak)

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[3] See, in particular, Rowthorn (1999), p. 418, equations (14) and (15).

[4] See, for example, Bean, Layard and Nickell (1987), Layard, Nickell, and Jackman (1991), Nickell (1995).

[5] In his empirical study he uses annual data from 1955 to 1989 for 17 OECD countries.

must be imposed on the long-run behavior of the entire general equilibrium system. After all, the steady state of this general equilibrium system must have the property that the unemployment rate is trendless and that the ratio of capital to labor (in efficiency units) is trendless as well. However, it turns out, as we will show in Section 3, that the invariance restrictions are not necessary in this context either. While it is true that some restrictions do need to be imposed on the general equilibrium system to guarantee that the unemployment is trendless in the long run, these restrictions are much weaker than even weak unemployment invariance.

Section 2 examines the invariance restrictions in the context of the basic model of Layard, Nickell and Jackman (1991). In Section 3 we present another simple model that highlights why unemployment invariance is an important issue for understanding and predicting labor market activity, as well as formulating policy. Section 4 extends this model and shows how long-run unemployment trendlessness arises not from labor market activity alone, but in conjunction with the activity in other markets. Section 5 provides an empirical illustration, comprising a model of the UK labor market, where we test for strong and weak invariance. Finally, Section 6 concludes.

2 Unemployment Invariance Restrictions

In much of the existing literature, the unemployment invariance restrictions are specified in terms of the Layard-Nickell-Jackman (LNJ) model. To set the stage, we now proceed to specify the restrictions in terms of this model.

The central LNJ model (specified in LNJ (1991, ch. 8) and summarized on p. 368)) consists of a price mark-up equation and a wage mark-up equation. These equations are meant to describe equilibrium unemployment as the outcome of the "battle of the mark-ups".

First, prices are set as a mark-up on wages:

$$P_t - W_t = \beta_0 - \beta_1 u_t - \beta_2 K_t + \beta_3 L_t \qquad (1)$$

where P_t is the price level, W_t is the wage level, u_t is the unemployment rate, K_t is the capital stock, L_t is the labor force, and the coefficients β_i ($i > 0$) are all positive. Following the standard set-up, all variables (except the unemployment rate) are in logs.

Second, wages are set as a mark-up on prices:

$$W_t - P_t = \gamma_0 - \gamma_1 u_t + \gamma_2 K_t - \gamma_3 L_t, \qquad (2)$$

where the coefficients γ_i ($i > 0$) are all positive.

Thus, by (1) and (2), the equilibrium unemployment rate is

$$u_t^* = \frac{\gamma_0 + \beta_0}{\beta_1 + \gamma_1} + \frac{\gamma_2 - \beta_2}{\beta_1 + \gamma_1} K_t + \frac{\beta_3 - \gamma_3}{\beta_1 + \gamma_1} L_t \tag{3}$$

In this context, the "strong invariance restrictions" - ensuring that the unemployment rate is independent of the capital stock and the labor force in the long run - are:

$$\gamma_2 = \beta_2, \quad \beta_3 = \gamma_3. \tag{4}$$

In fact, LNJ (p. 368) impose the restrictions $\beta_2 = \beta_3 = \gamma_2 = \gamma_3$, which are even more restrictive than the strong invariance restrictions (4).

Suppose that the capital-labor ratio ($K_t - L_t$) is trendless. Then the "weak invariance restriction" may be specified as

$$\gamma_2 - \beta_2 = -(\beta_3 - \gamma_3) \tag{5}$$

(Then, clearly, the equilibrium unemployment rate depends on the capital-labor ratio, but not on the individual magnitudes of capital and labor.)

3 Why Unemployment Invariance Matters

To see why the unemployment invariance hypothesis matters, let us consider another simple model of the labor market, corresponding to the one underlying Figure 1. This model is specified in terms of a labor demand function (corresponding to the $P - W$ markup equation above), a wage setting function (equivalent to the $W - P$ markup equation above), a labor supply function, and a definition of the unemployment rate.

3.1 A Model

Let the labor demand equation be

$$E_t = a_0 + a_E E_{t-1} - a_w w_t + a_K K_t + a_Z Z_t \tag{6}$$

describing firms' aggregate labor demand. E is employment, w is the real wage, K is the capital stock, and Z is the working-age population, and all coefficients are positive.[6]

[6] Current employment depends positively on lagged employment due to employment adjustment costs, inversely on the real wage due to diminishing returns to labor, positively on the capital stock due to capital-labor complementarities, and positively on population since more job seekers reduce search costs.

This equation represents the only substantial contrast between this model and the previous one. The first difference is that equation (6) describes the employment level, whereas the LNJ $P - W$ markup equation is in terms of the employment rate (employment as a fraction of the labor force). In particular, the LNJ $P - W$ markup equation can be expressed as the following labor demand equation: $E_t - L_t = -\frac{\beta_0}{\beta_1} - \frac{1}{\beta_1}(W_t - P_t) + \frac{\beta_2}{\beta_1}(K_t - L_t)$, where $E_t - L_t$ is the employment rate. However, equation (6) is more plausible: given that labor demand is the outcome of profit maximization subject to a production function, we expect the employment level - rather than the employment rate - to depend on the real wage and the capital-labor ratio.

The second difference is that equation (6) contains employment inertia (via the term $a_E E_{t-1}$). The LNJ model (1991, ch. 8) introduces dynamics via the wage equation; in particular, the dynamic version of the LNJ $W - P$ markup equation is $W_t - P_t = \gamma_0 - \gamma_1 u_t + \gamma_2 K_t - \gamma_3 L_t - \gamma_4 \Delta u_t$. As we will show below, this form of dynamics has markedly different implications from the employment inertia in (6).

Let the wage setting equation be

$$w_t = b_E E_t + b_K K_t \tag{7}$$

describing the outcome of a wage bargaining process.[7] (The differences between this wage setting equation and the one given in the section above are not matters of substance.) The labor force is

$$L_t = c_Z Z_t, \tag{8}$$

where L is labor supply. All variables (except the unemployment rate, below) are in logs. The unemployment rate is[8]

$$u_t = L_t - E_t. \tag{9}$$

Substituting equation (7) into (6), we obtain the following employment equation:

$$E_t = \alpha_0 + \alpha_E E_{t-1} + \alpha_K K_t + \alpha_Z Z_t, \tag{10}$$

where $\alpha_0 = \frac{a_0}{1+a_w b_E}$, $\alpha_E = \frac{a_E}{1+a_w b_E}$, $\alpha_K = \frac{a_K - a_w b_K}{1+a_w b_E}$, and $\alpha_Z = \frac{a_Z}{1+a_w b_E}$. This equation may be rewritten as follows:

$$E_t = -\left(\frac{\alpha_E}{1-\alpha_E}\right) \Delta E_t + \left(\frac{\alpha_K}{1-\alpha_E}\right) K_t + \left(\frac{\alpha_Z}{1-\alpha_E}\right) Z_t + \left(\frac{\alpha_0}{1-\alpha_E}\right). \tag{11}$$

[7] The real wage depends positively on the capital stock because the latter stimulates productivity (thereby increasing the firms' surplus in bargaining) and has an income effect on households (thereby reducing the workers' surplus in bargaining).

[8] This is an approximation.

Substituting the employment equation (11) and the labor supply equation (8) into (9), and letting employment rate in the steady state grow at the constant rate g_E, we obtain the following steady-state unemployment rate:[9]

$$u^{LR} = \left(c_Z - \frac{\alpha_Z}{1 - \alpha_E} \right) Z_t - \left(\frac{\alpha_K}{1 - \alpha_E} \right) K_t + \left(\frac{\alpha_E}{1 - \alpha_E} \right) g_E - \left(\frac{\alpha_0}{1 - \alpha_E} \right), \tag{12}$$

where the steady-state employment growth rate is[10]

$$g_E = \left(\frac{\alpha_K}{1 - \alpha_E} \right) g_K + \left(\frac{\alpha_Z}{1 - \alpha_E} \right) g_Z, \tag{13}$$

and g_K, g_Z are the steady-state values of the capital stock growth rate (ΔK_t) and population growth rate (ΔZ_t), respectively.

The term $\frac{\alpha_E}{1 - \alpha_E} g_E$ in the unemployment equation (12) represents "frictional growth", i.e. the interaction between employment growth and lagged employment adjustment. Since employment growth is positive and employment adjustment is characterized by inertia (given by the inertia coefficient α_E), employment is "chasing after a moving target", i.e. after the frictionless employment level. The greater is employment growth and the greater the inertia coefficient α_E, the further behind its target does actual employment fall, and thus the higher is unemployment. Thus observe that the effect of employment inertia does not ever die out in this model; its interaction with employment growth affects the long-run unemployment rate, and it also influences the way this unemployment rate depends on the capital stock and the labor force. Note that the phenomenon of frictional growth cannot be captured in the LNJ model above, since that model is specified solely in terms of stationary variables (the capital-labor ratio and the unemployment rate

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[9] Note that unemployment rate is given by

$$u_t = \left(c_Z - \frac{\alpha_Z}{1 - \alpha_E} \right) Z_t - \left(\frac{\alpha_K}{1 - \alpha_E} \right) K_t + \left(\frac{\alpha_E}{1 - \alpha_E} \right) \Delta E_t - \left(\frac{\alpha_0}{1 - \alpha_E} \right).$$

We assume that the growth rate of employment (ΔE_t) stabilizes to the constant g_E in the steady-state. For the various reasons to be given below, steady-state unemployment u^{LR} is trendless (lacks a time subscript) even though the population Z_t and the capital stock K_t grow through time.

[10] To see this take the first difference of eq. (10):

$$\Delta E_t = \alpha_E \Delta E_{t-1} + \alpha_K \Delta K_t + \alpha_Z \Delta Z_t.$$

In the steady-state we assume that all growth rates stabilize to some constants, i.e $\Delta E_t = \Delta E_{t-1} = g_E$, $\Delta K_t = g_K$, and $\Delta Z_t = g_Z$. Hence, we obtain eq. (13).

are assumed stationary in the long run), and thus the interaction between growth and labor market frictions does not occur.

In the context of our model, the strong invariance conditions are

$$c_Z = \frac{\alpha_Z}{1 - \alpha_E}, \quad \alpha_K = 0. \tag{14}$$

This implies that the steady state unemployment rate (12) becomes

$$u^{LR} = \left(\frac{\alpha_E}{1 - \alpha_E}\right) g_E - \left(\frac{\alpha_0}{1 - \alpha_E}\right),$$

i.e. the equilibrium of the labor market ensures that variations in population Z_t and the capital stock K_t have no influence on the steady-state unemployment rate.

Suppose that the capital-population ratio $(K_t - Z_t)$ is trendless. Then the weak invariance condition is

$$c_Z - \frac{\alpha_Z}{1 - \alpha_E} = \frac{\alpha_K}{1 - \alpha_E}. \tag{15}$$

Thus the steady state unemployment rate (12) becomes

$$u^{LR} = -\left(\frac{\alpha_Z}{1 - \alpha_E} - c_Z\right)(K_t - Z_t) + \left(\frac{\alpha_E}{1 - \alpha_E}\right) g_E - \left(\frac{\alpha_0}{1 - \alpha_E}\right),$$

and since $(K_t - Z_t)$ is trendless by assumption, the steady state unemployment rate is trendless as well.

In the absence of strong or weak invariance, the following condition is required to ensure that the unemployment rate is trendless in the long run:[11]

$$\left(c_Z - \frac{\alpha_Z}{1 - \alpha_E}\right) g_Z = \left(\frac{\alpha_K}{1 - \alpha_E}\right) g_K. \tag{16}$$

Here the equilibration mechanism, ensuring the trendlessness of steady-state unemployment, lies (in part)[12] outside the labor market. To fix ideas, let us assume that the population growth rate g_Z is exogenously given and that the capital stock adjusts so as to achieve a steady state in which capital accumulation g_K is constant and satisfies (16). (The Solow model provides an example of how the capital stock adjusts to achieve the steady state, given an exogenous rate of population growth.)

[11] This condition can be derived by taking the first difference of eq. (12):

$$\Delta u_t^{LR} = \left(c_Z - \frac{\alpha_Z}{1 - \alpha_E}\right) \Delta Z_t^{LR} - \left(\frac{\alpha_K}{1 - \alpha_E}\right) \Delta K_t^{LR}$$

and setting $\Delta u_t^{LR} = 0$ (i.e. the unemployment rate stabilizes to some constant in the steady-state).

[12] Of course some of the equilibration also occurs in the labor market, since employment responds to the capital stock and working age population.

3.2 Two Policy Exercises

In the context of this model, we now proceed to show why the unemployment invariance conditions matter by examining two comparative dynamic exercises. First, consider the response of (steady-state) unemployment to a policy that generates an upward shift of the path of the capital stock. Specifically, the capital stock increases by $K_t = K_t' - K_t$ in each period but its growth rate remains unchanged. The implications of this policy for unemployment are as follows:

- Under strong invariance, there is no effect on the unemployment (by (14)).
- Under weak invariance, the steady-state unemployment rate falls by $\left(\frac{\alpha_Z}{1-\alpha_E} - c_Z\right) dK_t$.
- In the absence of strong or weak invariance, the steady-state unemployment rate falls by $\left(\frac{\alpha_K}{1-\alpha_E}\right) dK_t$, where $\frac{\alpha_K}{1-\alpha_E}$ may of course differ from $\left(\frac{\alpha_Z}{1-\alpha_E} - c_Z\right)$.

In our second comparative dynamic exercise, consider the steady-state unemployment effect of a policy that increases labor force participation (e.g. job counselling, wage subsidies, payroll tax reductions). Specifically, the policy raises the coefficient α_Z in the employment equation.

- Under strong invariance, the increase in α_Z (denoted by $d\alpha_Z$) is matched by an increase in c_Z, namely $dc_Z = \left(\frac{1}{1-\alpha_E}\right) d\alpha_Z$, and thus the only influence on the steady-state unemployment rate is through the employment growth rate term in (12): $du_t^{LR} = \frac{\alpha_E g_Z}{(1-\alpha_E)^2} d\alpha_Z$. This influence is negligible for small population growth rates.
- Under weak invariance, the effect is the same as above, since $c_Z - \frac{\alpha_Z}{1-\alpha_E} = \alpha_K$ (by equation (15)) and the coefficient α_K remains unchanged.
- In the absence of strong or weak invariance, the growth rate of capital stock needs to adjust so that unemployment stabilizes (by eq. (16)). Thus the steady-state unemployment rate changes by[13]

$$du^{LR} = -\left(\frac{Z_t}{1-\alpha_E}\right) d\alpha_Z - \left(\frac{\alpha_K}{1-\alpha_E}\right) dK_t. \qquad (17)$$

[13] Comparing the two steady states, note that dK_t adjusts to Z_t so that the right-hand side of equation (17) is a constant. The underlying computations are given in Appendix B.

These examples show clearly how important the unemployment invariance conditions are for the predictions and policy implications of the model.

4 Growth, Unemployment Trendlessness, and Unemployment Invariance

We now extend the model above to an endogenous growth context and show that neither the weak nor strong forms of the unemployment invariance hypothesis, applied to the labor market, are necessary to ensure the long-run trendlessness of unemployment.

4.1 A Model

Our model is in the spirit of Romer (1990), Grossman and Helpman (1991), and Aghion and Howitt (1992), but we develop it as an extention of the models above.

In the Solow growth model, goods can be used for two purposes, consumption and investment. We now assume that they can serve a third purpose as well: R&D. For analytical simplicity (but without loss of generality),[14] assume that R&D is produced through labor alone. Let E_t^{τ} be the amount of labor in the R&D sector, whose output is the rate of technological progress $\Delta\tau_t$. Let the production function of the R&D sector be

$$\Delta\tau_t = \frac{1}{\theta} E_t^{\tau}. \tag{18}$$

The more labor is devoted to R&D, the less is available for the production of consumption and investment goods and the faster the rate of technological progress.

As in the models above, let E_t be the amount of labor devoted to consumption and investment goods. Then the unemployment rate becomes

$$u_t = L_t - E_t - \theta\Delta\tau_t. \tag{19}$$

Now let the price markup over the wage be

$$P_t - W_t = \beta_0 + \beta_E E_t - \beta_L L_t - \beta_K K_t - \beta_\tau \tau_t, \tag{20}$$

where τ_t is a technological variable (and $\Delta\tau_t$ is technological change); and let the wage markup over the price be

[14] It is straightforward to extend the model to let R&D be generated by both labor and capital.

$$W_t - P_t = \gamma_0 - \gamma_u u_t + \gamma_K K_t + \gamma_\tau \tau_t, \tag{21}$$

Then, by (20), (21), and (19), the labor market equilibrium condition now becomes

$$u_t^* = \frac{(\beta_0 + \gamma_0) + (\beta_E - \beta_L) L_t + (\gamma_K - \beta_K) K_t + (\gamma_\tau - \beta_\tau) \tau_t - \beta_E \theta \Delta \tau_t}{\gamma_u + \beta_E}. \tag{22}$$

Next, consider the capital goods market. Let V_t be the nominal user cost of capital. Let the price mark-up over the user cost depend positively on the capital stock K_t (due to diminishing returns to capital), negatively on employment E_t (since a rise in employment is assumed to increase the productivity of capital, permitting the firm to reduce its price relative to the user cost), and negatively on the technological variable τ_t (since a rise in τ_t also raises the productivity of capital):

$$P_t - V_t = \alpha_0 + \alpha_K K_t - \alpha_E E_t - \alpha_\tau \tau_t, \tag{23}$$

where the parameters $\alpha_0, \alpha_E, \alpha_K,$ and α_τ are positive.

Furthermore, let the user cost mark-up over the price depend negatively on the capital stock K_t (since an increase in capital stock reduces the cost of producing new capital goods, i.e. investment), and positively on employment E_t and the technological variable τ_t (since an increase in E_t and τ_t raise productivity and enable firms to demand a higher user cost):

$$V_t - P_t = \delta_0 - \delta_K K_t + \delta_E E_t + \delta_\tau \tau_t, \tag{24}$$

where $\delta_0, \delta_I, \delta_K, \delta_E, \delta_\tau > 0$.

By equations (23), (24), and (19), the capital goods market equilibrium condition becomes

$$u_t^* = \frac{(\alpha_E - \delta_E) L_t - (\alpha_0 + \delta_0) - (\alpha_K - \delta_K) K_t}{\alpha_E - \delta_E} + \frac{(\alpha_\tau - \delta_\tau) \tau_t - (\alpha_E - \delta_E) \theta \Delta \tau_t}{\alpha_E - \delta_E}. \tag{25}$$

We may rewrite the labor market equilibrium and capital goods market equilibriium conditions as

$$u_t^* = a_0 + a_L L_t + a_K K_t + a_\tau \tau_t + a_g \Delta \tau_t, \tag{26}$$

$$u_t^* = b_0 + b_K K_t + L_t + b_\tau \tau_t + b_g \Delta \tau_t, \tag{27}$$

where $a_0 = \frac{\beta_0 + \gamma_0}{\gamma_u + \beta_E}$, $a_L = \frac{\beta_E - \beta_L}{\gamma_u + \beta_E}$, $a_K = \frac{\gamma_K - \beta_K}{\gamma_u + \beta_E}$, $a_\tau = \frac{\gamma_\tau - \beta_\tau}{\gamma_u + \beta_E}$; and $b_0 = -\frac{\alpha_0 + \delta_0}{\alpha_E - \delta_E}$, $b_K = \frac{\delta_K - \alpha_K}{\alpha_E - \delta_E}$, $b_\tau = \frac{\alpha_\tau - \delta_\tau}{\alpha_E - \delta_E}$, $a_g = \frac{-\beta_E \theta}{\gamma_u + \beta_E}$ and $b_g = -\theta$.

4.2 Invariance Conditions

By equation (26), the strong invariance restrictions, applied to the labor market, may be expressed as follows:

$$a_L = 0, \; a_K = 0, \; \text{and } a_\tau = 0. \tag{28}$$

The weak invariance restrictions, applied to the labor market, ensure that the behavior of the labor market is such that the ratio of capital to labor in efficiency units $\frac{\tilde{K}_t}{\tilde{L}_t \tilde{\tau}_t}$ is constant[15]. For our model, these restrictions are:

$$a_K = -a_L = -a_\tau \tag{29}$$

Beyond that, there is one further set of invariance restrictions encountered in the literature, namely, weak invariance applied to the general equilibrium system (rather than just to the labor market). These restrictions on the parameters ensure that the behavior of the general equilibrium system is such that the ratio of capital to labor in efficiency units $\frac{\tilde{K}_t}{\tilde{L}_t \tilde{\tau}_t}$ is constant. This property - which may be called "weak general equilibrium invariance" - is present, for example, in Rowthorn (1999, p. 412, equation (20)). In the Rowthorn model, the constancy of the long-run unemployment rate requires the constancy of the long-run ratio of capital to labor in efficiency units.

4.3 Balanced Growth in the Absence of the Invariance Restrictions

We now return to our model and suppose that the equilibration process, ensuring balanced growth with a trendless unemployment rate, occurs not only in the labor market but also through adjustments in capital accumulation and R&D, as is standard in a variety of endogenous growth models. In particular, capital accumulation and R&D adjust so that the economy approaches a steady state in which (a) the labor market and the capital goods market are both in equilibrium, (b) the capital stock grows at a constant rate, (c) the rate of technological progress is constant and (d) the unemployment rate is constant.

Furthermore, taking first differences of equations (26) and (27), and recalling that the unemployment rate is constant in the long run, we obtain the long-run equilibrium growth rates of the capital stock (g_K) and technological

[15] The "~" above the variable denotes its level.

progress (g_τ) as a function of the growth rate of labor force (g_L):[16]

$$g_K = \left(\frac{a_\tau - a_L b_\tau}{b_\tau a_K - a_\tau b_K} \right) g_L, \tag{30}$$

$$g_\tau = \left(\frac{-a_K + a_L b_K}{b_\tau a_K - a_\tau b_K} \right) g_L. \tag{31}$$

These conditions ensure that unemployment is trendless.

Equations (30) and (31) imply the following relationships between the levels of capital stock $\left(\tilde{K}_t\right)$, technological progress ($\tilde{\tau}_t$), and labor force $\left(\tilde{L}_t\right)$:[17]

$$\tilde{K}_t = A \tilde{L}_t^{\alpha}, \tag{32}$$

$$\tilde{\tau}_t = C \tilde{L}_t^{\beta}, \tag{33}$$

respectively; and where $\alpha = \frac{a_\tau - a_L b_\tau}{b_\tau a_K - a_\tau b_K}$, $\beta = \frac{-a_K + a_L b_K}{b_\tau a_K - a_\tau b_K}$, and A, C are arbitrary constants. Manipulation of the above equations shows that the capital labor ratio in efficiency units is not constant over time:

$$\frac{\tilde{K}_t}{\tilde{L}_t \tilde{\tau}_t} = \frac{A}{C} \tilde{L}_t^{\alpha - \beta - 1}. \tag{34}$$

Observe that this condition is of course weaker than even weak general equilibrium invariance. Recall that under general equilibrium invariance, the ratio of capital to labor in efficiency units ($\frac{\tilde{K}_t}{\tilde{L}_t \tilde{\tau}_t}$) is constant. This special case holds only under the additional restriction that $\alpha = 1 + \beta$. However, as shown, this condition is not required to ensure unemployment trendlessness and balanced growth.

5 Empirical Illustration

Using an autoregressive distributed lag (ARDL) approach to cointegration analysis,[18] we estimate the following UK labor market model with annual

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[16] First differencing of (26) and (27) gives:

$$\Delta u^* = a_L g_L + a_K g_K - a_\tau g_\tau = 0,$$
$$\Delta u^* = g_L - b_K g_K - b_\tau g_\tau = 0,$$

respectively. Solving these equations, for a given growth rate of the labor force, we obtain (30) and (31).

[17] Note that $\log \tilde{K}_t \equiv K_t$, $\log \tilde{\tau}_t \equiv \tau_t$, $\log \tilde{L}_t \equiv L_t$.

[18] This approach has been developed by Pesaran and Shin (1995), Pesaran (1997), and Pesaran et al. (1996). We refer the reader to Henry, Karanassou, and Snower (2000) for further details of the estimation.

data for the period 1964-1997:

$$\Delta E_t = \underset{(1.05)}{3.16} - \underset{(0.05)}{0.31} E_{t-2} - \underset{(0.05)}{0.09} w_t + \underset{(0.03)}{0.14} K_t + \underset{(0.38)}{3.04} \Delta K_t - \underset{(0.32)}{1.98} \Delta K_{t-1} - \underset{(0.13)}{0.51} TR_t - \underset{(0.001)}{0.01} p_t^{oil}, \quad (35)$$

$$\Delta w_t = -\underset{(0.11)}{0.34} - \underset{(0.07)}{0.31} w_{t-2} + \underset{(0.04)}{0.16} b_t - \underset{(0.34)}{1.18} \Delta TR_t - \underset{(0.14)}{0.50} u_t, \quad (36)$$

$$\Delta L_t = -\underset{(0.02)}{0.004} + \underset{(0.12)}{0.41} \Delta L_{t-1} - \underset{(0.07)}{0.25} L_{t-2} - \underset{(0.07)}{0.16} \Delta u_t + \underset{(0.01)}{0.02} w_t + \underset{(*)}{0.25} Z_t, \quad (37)$$

where Δ is the difference operator, standard errors are in parentheses[19], and the definitions of the variables are given below:

E_t = log of employment, L_t = log of labour force,
u_t = unemployment rate $(u_t = L_t - E_t)$,
w_t = log of real compensation per person employed,
K_t = log of real capital stock, p_t^{oil} = log of real oil price,
b_t = log of real social security benefits per person,
TR_t = indirect taxes as % of GDP, Z_t = log of working age population.

Note that the above estimated equations were selected on the basis of the Akaike Information Criterion or the Schwarz Bayesian Criterion, and consist of stationary, well-specified linear combinations of the variables involved.[20] The figure below shows that our estimated model tracks the data very well.

[19] The (*) in equation (37) indicates that we have restricted the long-run elasticity of population to unity. This can be justified as follows. The unrestricted version of equation (37)

$$\Delta L_t = -\underset{(1.08)}{0.50} + \underset{(0.12)}{0.40} \Delta L_{t-1} - \underset{(0.07)}{0.26} L_{t-2} + \underset{(0.10)}{0.29} Z_t - \underset{(0.07)}{0.16} \Delta u_t + \underset{(0.01)}{0.01} w_t$$

shows that the coefficient of Z_t is (i) statistically significant at any conventional level, and (ii) very close in magnitude, and with opposite sign, to the parameter of L_{t-2}. These observations led us to impose the restriction that the long-run elasticity of population is unity which could not be rejected at any conventional size of the test.

[20] In addition to the standard misspecification tests (linearity, no serial correlation, homoskedasticity, and normality), all equations pass the CUSUM and CUSUMSQ tests for structural stability. (The least squares estimation and the diagnostic tests are given in the Appendix A.) Our preferred specifications (35)-(37) have been estimated using 3SLS in order to take into account potential endogeneity and cross equation correlation.

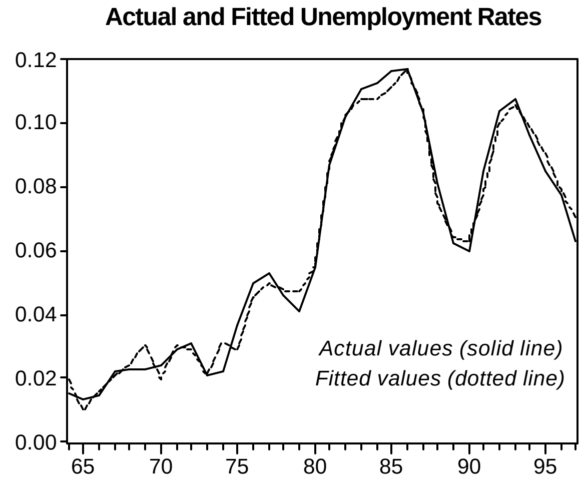

Figure 2

For expositional purposes, we rewrite the labor market system (35)-(37) as

$$
\begin{aligned}
E_t &= \beta_1 + E_{t-1} - \beta_2 E_{t-2} - \beta_3 w_t + \beta_4 K_t + \beta_5 \Delta K_t - \beta_6 \Delta K_{t-1} \\
&\quad - \beta_7 TR_t - \beta_8 p_t^{oil}, \qquad (38)\\
w_t &= \beta_9 + w_{t-1} - \beta_{10} w_{t-2} + \beta_{11} b_t - \beta_{12} \Delta TR_t - \beta_{13} u_t, \qquad (39)\\
L_t &= \beta_{14} + (1+\beta_{15}) L_{t-1} - (\beta_{15}+\beta_{16}) L_{t-2} - \beta_{17} \Delta u_t + \beta_{18} w_t + \beta_{16} Z_t, \qquad (40)
\end{aligned}
$$

where all the β's are positive. Using equations (38)-(40) together with the definition of the unemployment rate, $u_t = L_t - E_t$, Henry, Karanassou and Snower (2000) derive the "reduced form" unemployment rate equation and its long-run solution[21] $\left(u_t^{LR}\right)$:

$$
\begin{aligned}
\lambda u_t^{LR} &= \beta_2\beta_{10}(\beta_{14} + \beta_{16} Z_t) + (\beta_{16}\beta_3 + \beta_2\beta_{18})(\beta_9 + \beta_{11} b_t - \beta_{12} \Delta TR_t) \\
&\quad - \beta_{16}\beta_{10}\left(\beta_1 + \beta_4 K_t + \beta_5 \Delta K_t - \beta_6 \Delta K_{t-1} - \beta_7 TR_t - \beta_8 p_t^{oil}\right), \qquad (41)
\end{aligned}
$$

where $\lambda = \beta_2\beta_{18}\beta_{13} + \beta_{16}\beta_3\beta_{13} + \beta_2\beta_{10}\beta_{16}$.

5.1 Testing the Strong Invariance Restrictions

According to the strong restrictions, the long-run unemployment rate should not depend on growing variables like capital stock (K_t) and working age

[21] The existence of a long-run unemployment rate equation is guaranteed by the dynamic stability of the labour market system (35)-(37).

population (Z_t). Therefore, in the context of eq. (41), we can express the strong invariance restrictions as follows:

$$H_K \;:\; \beta_{16}\beta_{10}\beta_4 = 0,$$
$$H_Z \;:\; \beta_2\beta_{10}\beta_{16} = 0.$$

Observe that, in the context of our model, the above null hypotheses cannot hold unless any of the individual parameters involved is zero, i.e.

$$H_K^0 \;:\; \beta_4 = 0 \text{ or } \beta_{10} = 0 \text{ or } \beta_{16} = 0,$$
$$H_Z^0 \;:\; \beta_2 = 0 \text{ or } \beta_{10} = 0 \text{ or } \beta_{16} = 0.$$

The reasoning is straightforward. Our unemployment rate equation (41) is derived by subtracting (the long-run solution of) labor demand from labor supply (having …rst substituted out in these equations the wage setting function). Therefore, when labor demand does not depend on capital stock $(\beta_4 = 0)$ the unemployment rate will not depend on capital stock either and the strong invariance condition will be valid. On the other hand, if $\beta_{16} = 0$ then the labor supply equation (40) is not dynamically stable.[22] In this case we cannot even derive a reduced form unemployment rate equation, let alone testing any restrictions. A similar situation arises when the wage setting equation (39) is not dynamically stable, i.e. $\beta_{10} = 0$, or when the labor demand equation (38) is not stable $(\beta_2 = 0)$.

However, we should make clear that the above reasoning does not imply that individual testing is the same as joint testing of the parameters.[23] We only argue that the hypothesis H_K (H_Z) can be substantiated if any of the individual hypotheses in H_K^0 (H_Z^0) cannot be rejected. It is not di¢cult to see that our estimations reject the above hypotheses, since all the coe¢cients of the labor market system (35)-(37) are statistically di¤erent from zero at any conventional signi…cance level.

We can also test the strong invariance restrictions in the standard integration - cointegration framework as opposed to the ARDL technique followed by Henry et al. (2000). Consider the following error correction form of the

[22] Note that dynamic stability of an equation implies that the variables involved are cointegrated.

[23] Generally, individual and joint testing will yield di¤erent results due to the non-zero covariances between the parameters.

autoregressive distributed lag equations (38)-(40):

$$\begin{aligned}
\Delta E_t &= \beta_1 + \beta_2 \Delta E_{t-1} - \beta_3 \Delta w_t + (\beta_5 - \beta_4)\,\Delta K_t \\
&\quad - \beta_6 \Delta K_{t-1} - \beta_7 \Delta TR_t - \beta_8 \Delta p_t^{oil} \\
&\quad - \beta_2 \left(E_{t-1} + \frac{\beta_3}{\beta_2} w_{t-1} - \frac{\beta_4}{\beta_2} K_{t-1} + \frac{\beta_7}{\beta_2} TR_{t-1} + \frac{\beta_8}{\beta_2} p_t^{oil} \right), \qquad (42)\\
\Delta w_t &= -\beta_9 + \beta_{10} \Delta w_{t-1} + \beta_{11} \Delta b_t - \beta_{12} \Delta TR_t - \beta_{13} \Delta u_t \\
&\quad - \beta_{10} \left(w_{t-1} - \frac{\beta_{11}}{\beta_{10}} b_{t-1} + \frac{\beta_{13}}{\beta_{10}} u_{t-1} \right), \qquad (43)\\
\Delta L_t &= -\beta_{14} + (\beta_{15} + \beta_{16})\,\Delta L_{t-1} - \beta_{17} \Delta u_t + \beta_{18} \Delta w_t + \beta_{16} \Delta Z_t \\
&\quad - \beta_{16} \left(L_{t-1} - \frac{\beta_{18}}{\beta_{16}} w_{t-1} - Z_{t-1} \right), \qquad (44)
\end{aligned}$$

respectively. Therefore, the long-run solutions of (38)-(40),

$$\left[E_t - \frac{1}{\beta_2} \left(-\beta_3 w_t + \beta_4 K_t - \beta_7 TR_t - \beta_8 p_t^{oil} \right) \right], \qquad (45)$$

$$\left[w_t - \frac{1}{\beta_{10}} \left(\beta_{11} b_t - \beta_{13} u_t \right) \right], \qquad (46)$$

$$\left[L_t - \frac{1}{\beta_{16}} \left(\beta_{18} w_t + \beta_{16} Z_t \right) \right], \qquad (47)$$

should represent cointegrating vectors.

Now recall that the strong invariance restrictions require that either $\beta_{10} = 0$, or $\beta_{16} = 0$, or $\beta_2 = \beta_4 = 0$. Therefore, non-cointegration of the variables in any of the three vectors (45)-(47) would provide evidence for the validity of the strong invariance restrictions. However, when we test for cointegration using the Johansen procedure, we cannot reject the hypotheses that the above linear combinations of variables are stationary.[24] In other

[24] In particular, the evidence from the Johansen procedure is that the variables involved in each of the equations (45)-(47) are cointegrated. For example, using the maximal eigenvalue and trace statistics of the Johansen procedure, we find that there is a cointegrating vector among the variables involved in the labor demand equation: $E_t, w_t, K_t, TR_t, p_t^{oil}$. We do not report these tests to save space.

Furthermore, using likelihood ratio statistics we test whether the coefficients of these cointegrating vectors conform with our ARDL estimations. The likelihood ratio tests for the restrictions imposed on the cointegrating vectors (45)-(47) are:

$$\begin{aligned}
\chi^2(4) &= 8.27\ [0.08], \\
\chi^2(2) &= 7.24\ [0.03], \\
\chi^2(3) &= 8.27\ [0.34],
\end{aligned}$$

words, the results obtained from the Johansen procedure validate our conclusion from the ARDL approach that the strong invariance restrictions are rejected.

5.2 Testing for Weak Invariance

According to the weak invariance hypothesis, the long-run unemployment rate can be a function of the ratios of growing variables. In the context of our empirical model, weak invariance implies that the long run unemployment depends on the ratio of population to the capital stock. That is, on the right-hand side of eq. (41) we should have the difference between the log of population and the log of capital stock ($Z_t - K_t$) as an explanatory variable. So weak invariance requires the following restriction on the long-run unemployment equation (41):

$$H_{K,Z} : \beta_2 = \beta_4.$$

Imposing the above null hypothesis on eq. (41) gives

$$\begin{aligned} \lambda u_t^{LR} = \; & \beta_2\beta_{10}\beta_{16}(Z_t - K_t) + (\beta_{16}\beta_3 + \beta_2\beta_{18})(\beta_9 + \beta_{11}b_t - \beta_{12}\Delta TR_t) \\ & + \beta_2\beta_{10}\beta_{14} - \beta_{16}\beta_{10}\left(\beta_1 + \beta_5\Delta K_t - \beta_6\Delta K_{t-1} - \beta_7 TR_t - \beta_8 p_t^{oil}\right), \end{aligned} \tag{48}$$

It can be seen from eq. (38)-(40) that the $H_{K,Z}$ restriction only involves the parameters of the labor demand equation. In particular, weak invariance requires that the long-run elasticity of employment with respect to capital is unity. Using a Wald test, we find that the weak invariance hypothesis is clearly rejected by the data.[25]

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repsectively. (P-values are given in square brackets.)

For example, the likelihood ratio statistic $\chi^2(4) = 8.27\,[0.08]$ tests whether the estimated linear combination (45) of the variables involved in our labor demand equation is stationary:

$$E_t - \frac{1}{\beta_2}\left(-\beta_3 w_t + \beta_4 K_t - \beta_7 TR_t - \beta_8 p_t^{oil}\right),$$

where the β's are the estimates of the ARDL approach.

At conventional significance levels, the above tests cannot reject the null that the long-run relationships estimated using the ARDL approach do indeed represent cointegrating vectors.

[25] The test follows a $\chi^2(1)$ distribution, the value of the Wald statistic is 9.46, and the 5% critical value is 3.84.

6 Conclusion

In sum, this paper has argued that the weak and strong invariance restrictions, applied to the labor market, are unnecessary to ensure that the long-run unemployment rate is independent of the capital stock, the labor force, and productivity. There is no reason to believe that the labor market alone is responsible for ensuring that unemployment is trendless over the long run. In general, equilibrating mechanisms in the labor market and other markets are jointly responsible for this phenomenon. Thus the invariance restrictions need not be imposed on the specifications of labor market systems (such as the price mark-up and wage mark-up equations above), or on estimations of single-equation unemployment models. Restrictions on the relationships between the long-run growth rates of the capital stock, the labor force and technology are sufficient for this purpose.

We have shown that this result has important implications for prediction and policy. For example, imposing strong invariance restrictions on labor market activity implies that policies that shift upward the time path of the capital stock have no long-run effect on the unemployment rate; but if these restrictions are removed, these policies may have a permanent unemployment effect. Along analogous lines, removing such restrictions also implies that policies affecting labor force participation may influence the long-run unemployment rate as well.

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7 Appendix A

Table A1: OLS, 1964-1997

[1]	$\Delta E_t =$	2.87	$-0.31E_{t-2}$	$-0.10w_t$	$+0.15K_t$	
		(1.22)	(0.06)	(0.05)	(0.04)	
		$+3.13\Delta K_t$	$-2.02\Delta K_{t-1}$	$-0.54TR_t$	$-0.01p_t^{oil}$,	$R^2 = 0.89$
		(0.46)	(0.40)	(0.15)	(0.002)	
[2]	$\Delta w_t =$	-0.34	$-0.29w_{t-2}$	$+0.15b_t$	$-0.45u_t$	
		(0.11)	(0.08)	(0.04)	(0.14)	
		$-1.15\Delta TR_t$,				$R^2 = 0.50$
		(0.37)				
[3]	$\Delta L_t =$	-0.003	$+0.38\Delta L_{t-1}$	$-0.25L_{t-2}$	$-0.15\Delta u_t$	
		(0.02)	(0.13)	(0.07)	(0.07)	
		$+0.02w_t$,	$+0.25Z_t$,			$R^2 = 0.57$
		(0.01)	(*)			

(standard errors in parentheses)

* The restriction that the long-run elasticity of population is unity (coef.of $Z_t = -$ coef.of L_{t-2}) cannot be rejected at the 5% size of the test

Table A2: Misspecification tests

Equation	[1]	[2]	[3]
SC $[\chi^2(1)]$	1.33	0.25	0.28
LIN $[\chi^2(1)]$	0.33	0.23	0.67
NOR $[\chi^2(2)]$	0.28	1.43	0.73
HET $[\chi^2(1)]$	1.17	1.09	0.82
ARCH $[\chi^2(1)]$	0.59	1.09	0.29

5% critical values: $\chi^2(1) = 3.84$, $\chi^2(2) = 5.99$.

8 Appendix B

By (12), (13), and (16), we find that the change in steady-state unemployment is

$$du^{LR} = -\left(\frac{Z_t}{1-\alpha_E}\right) d\alpha_Z + \left(\frac{\alpha_E g_Z}{(1-\alpha_E)^2}\right) d\alpha_Z + \left(\frac{\alpha_E \alpha_K}{(1-\alpha_E)^2}\right) dg_K - \left(\frac{\alpha_K}{1-\alpha_E}\right) dK_t. \qquad (49)$$

The first term on the right-hand side of the above equation measures the effect that a change in α_Z, $d\alpha_Z$, has on the steady-state unemployment via the level of population; the second term describes how α_Z influences unemployment through employment growth; the third term captures the effect of the change in the capital stock growth rate (dg_K) on the steady-state unemployment; the last term measures the effect of the change in capital stock (which results from the change in its growth rate), dK_t, on the steady-state unemployment rate.

The relationship between $d\alpha_Z$ and dg_K is obtained by applying the differential operator $d(\cdot)$ on both sides of condition (16) and recalling that the only parameters that change are α_Z and g_Z:

$$dg_K = -\left(\frac{g_Z}{\alpha_K}\right) d\alpha_Z.$$

Substitution of the above into (49) gives eq. (17) in Section 2:

$$du^{LR} = -\left(\frac{Z_t}{1-\alpha_E}\right) d\alpha_Z - \left(\frac{\alpha_K}{1-\alpha_E}\right) dK_t. \tag{50}$$

We now show why the effect of a change in α_Z on the steady-state unemployment rate is constant through time, i.e. $du^{LR}_{t+1} = du^{LR}_t = du^{LR}$. Note that $d\alpha_Z$ denotes the change in α_Z: $d\alpha_Z \equiv \alpha'_Z - \alpha_Z$. Similarly, we have that $dK_t \equiv K'_t - K_t$, and $dg_K \equiv g'_K - g_K$. By (50) the change in the steady state unemployment, at period t, is

$$du^{LR}_t = -\left(\frac{Z_t}{1-\alpha_E}\right)(\alpha'_Z - \alpha_Z) - \left(\frac{\alpha_K}{1-\alpha_E}\right)(K'_t - K_t),$$

and

$$du^{LR}_{t+1} = -\left(\frac{Z_{t+1}}{1-\alpha_E}\right)(\alpha'_Z - \alpha_Z) - \left(\frac{\alpha_K}{1-\alpha_E}\right)\left(K'_{t+1} - K_{t+1}\right),$$

at period $t+1$. Subtracting the latter equation from the former gives

$$\Delta\left(du^{LR}_{t+1}\right) = \frac{-(\alpha'_Z - \alpha_Z)\,g_Z - \alpha_K g'_K + \alpha_K g_K}{(1-\alpha_E)} = 0,$$

since by the condition for a trendless steady-state unemployment rate (16) we have that

$$c_Z - \left(\frac{\alpha_Z}{1-\alpha_E}\right) g_Z = \left(\frac{\alpha_K}{1-\alpha_E}\right) g_K \text{ and}$$

$$c_Z - \left(\frac{\alpha'_Z}{1-\alpha_E}\right) g_Z = \left(\frac{\alpha_K}{1-\alpha_E}\right) g'_K.$$

Due to the constancy of du^{LR} over time, we omit its time subscript from the equation given in Section 2.