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On-the-Job Learning, Firing Costs and Employment

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Abstract

This paper explores the influence of on-the-job learning on the employment effect of firing costs. In the absence of on-the-job learning, the theoretical literature shows that firing costs may increase average employment (over the booms and recessions of the business cycle). We show that the existence of on-the-job learning weakens this effect. In fact, when the amount of on-the-job learning is sufficiently large, a rise in firing costs tends to reduce average employment.

KEYWORDS: Average employment; Firing costs; Severance Payments; On-the-job learning; Insider wage

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1 Introduction

The effect of firing costs on employment has been widely studied in the labor economics literature. This paper is concerned with how learning influences this effect. We focus on learning that arises on the job (as an automatic by-product of working), generating firm-specific skills. We show that as a result of the higher productivity arising from this learning, firing costs can have an adverse effect on average employment (over the booms and recessions of the business cycle).

This is an important issue because, as is well known, the supply of skilled workers and skilled jobs has risen dramatically, both in absolute terms and relative to unskilled workers and jobs, throughout the OECD over the last three decades. The importance of learning on the job has grown apace. If on-the-job learning has an important influence on the way firing costs affect employment, then the role of firing costs in the labor markets of advanced industrialized countries is undergoing change. Specifically, our analysis suggests that skill-biased technological change, applied to firm-specific skills, may cause firing costs to have a contractionary influence on employment. As is well known, most OECD countries have experienced pronounced skill-biased technological change over the past three decades, and during this time continental European countries with relatively restrictive job security legislation have, on average, been relatively unsuccessful at creating employment. Our analysis suggests a connection between these two empirical regularities.

The mainstream literature[1] explains how firing costs discourage both firing and hiring. Since firms that are firing must pay firing costs now whereas firms that are hiring may have to pay firing costs in the future, a standard result is that firing costs may discourage firing more than hiring, thereby raising employment. Other well-known factors pull in the opposite direction.[2] This paper contributes to this literature by showing that on-the-job learning may cause firing costs to have a less positive or even a negative influence on average employment. Intuitively, there are two main reasons:

1. In a downturn (when the firm is engaged in firing), firing costs raise

[1] See, for example, Bentolila and Bertola (1990) and Bertola (1990).

[2] These factors include the following: the rate at which the marginal product of labor declines (Bertola, 1992), firm heterogeneity (Bentolila and Saint-Paul, 1994), the probability of discontinuous drops in macroeconomic activity and the trend rate of productivity growth (Chen, Snower and Zoega, 2003), and wage effects of firing costs (Díaz-Vázquez and Snower, 1996).

employment *in efficiency units*; and in an upturn (when the firm is engaged in hiring), firing costs reduce employment *in efficiency units*. On account of on-the-job learning, the workers that are in the firm in the downturn are usually more productive than the workers hired in an upturn. Thus, one efficiency unit of labor in the downturn represents less workers than one efficiency unit of labor in the upturn. As a consequence, the resulting rise in the *number of people employed* in the recession may be smaller than the resulting fall in the *number of people employed* in the boom. In this sense, firing costs increase average employment to a lesser extent and may even reduce it.[3]

2. Since firing costs encourage firms to retain their incumbent workers in a downturn, these firms will need to hire fewer workers in the next recovery. Because the incumbent workers are more productive than the new workers, for each worker retained in the downturn, several new workers are not hired in the next recovery. In other words, the average productivity of the workforce is higher than it would otherwise be, and as a consequence, a smaller number of workers on average is required to produce a given cyclical trajectory of output. Again, firing costs increase average employment to a lesser extent and may even reduce it.

On-the-job learning causes firing costs to have a less positive or a negative influence on average employment even if firing costs affect employment via wages.[4] This occurs whether firing costs are severance payments or other costs.

It is also possible that, in the absence of learning, firing costs have a negative influence on average employment, i.e. firing costs discourage hiring more than firing.[5] This occurs when the rise in firing costs increases the

[3] It is clear that, in practice, the importance of this first channel is likely to depend on the length and depth of the recessions and booms. For instance, if the recession is short and shallow, there may well be labor hoarding. In that case workers are likely to be less productive in recessions than in booms, so that the opposite of this channel would apply. But if the recession is prolonged and deep, so that there is no labor hoarding - as is assumed in our analysis - this first channel becomes operative.

[4] As the literature indicates, a rise in firing costs increases the power of the incumbent workers or insiders in the wage negotiation (see Lindbeck and Snower, 1989), and also affects the new workers' wage (see Lazear, 1990, Bertola, 1990, and Mortensen and Pissarides, 1999). This, in turn, affects employment (see Díaz-Vázquez and Snower, 2003, 2005).

[5] When entrants receive their reservation wages, this can only occur when firing costs are not a transfer of income to the worker, e.g. trial costs (as is shown in the paper).

insider wage so much that the incentive for the firm to retain additional insiders in the recession is small. In this context, the existence of on-the-job learning may play a different role. Since the firm expects the new workers to learn on the job and become more productive in the future, it may not be so reluctant to hire them in the upturn. As a consequence, the negative influence of firing costs on hiring is weaker, and therefore the negative influence on average employment is also weaker.

To keep the analysis simple, we make some straightforward assumptions: all firms are alike (so that their behavior may be summarized by that of a representative firm), there is no employment growth, the labor demand function is linear and there is no labor hoarding. These assumptions are all harmless, since the effects of relaxing them are all well known. The influence of firing costs on employment via firm heterogeneity has been studied by Bentolila and Saint-Paul (1994), among others. The influence of employment growth on the employment effect of firing costs is examined in Chen, Snower, and Zoega (2003). The influence of the curvature of the production function (implying a nonlinear labor demand function) has been covered in Bertola (1992). All these influences may be super-imposed on our model, generating the expected modifications of our qualitative results. Thus, for brevity, we omit these influences in our analysis.

The paper is organized as follows. Section 2 presents the underlying model. Section 3 assumes exogenous wages and shows the main reasons why the existence of on-the-job learning influences the employment effect of firing costs. Section 4 considers endogenous wages. Section 5 concludes.

2 The model

2.1 Underlying assumptions

Consider a given number of identical firms (perfectly competitive in the product market[6]). Each firm has the following production function:[7]

$$Z_t E_t - \frac{b}{2} \left(E_t \right)^2 \tag{1}$$

[6]This assumption has no implications for the qualitative results of this paper.

[7]We assume a production function with linear marginal product of labor, which allows us to present the argument in the simplest form. In Appendix F we comment on the implications of considering a production function with nonlinear marginal product of labor.

where Z_t is a stochastic variable indexing business conditions, E_t is employment in efficiency units of labor and b is a positive constant. A two-state Markov chain describes the evolution of Z_t: in the "boom" $Z_t = Z^+$, and in the "recession" $Z_t = Z^-$, where Z^+ and Z^- are positive constants and $Z^+ > Z^-$. P is the probability of remaining in previous economic conditions, and thus $(1 - P)$ is the probability of a change in economic conditions. The values of Z^+ and Z^- determine the employment decision of the firm (for given parameters in the model). Specifically, we assume that the values of Z^+ and Z^- are such that in an upturn (when Z moves from a recession Z^- to a boom Z^+) the firm hires new workers, in a downturn (when Z moves from a boom Z^+ to a recession Z^-) the firm fires some workers, and when the current state (the boom or recession) persists, the firm retains its existing workers and hires no new ones.

We assume that n_t^+ is the number of new recruits that the firm hires in the upturn, who have no firm-specific skills. During their first period in the firm, the new recruits acquire these firm-specific skills through their learning on the job, so that at the end of the period they are incumbent workers with productivity $A > 1$.[8] We assume that in the downturn the firm fires the n^+ workers hired in the previous upturn, and retains a number of incumbent workers N_t^-. Thus, in the stationary equilibrium, the firm has N_t^- workers in the recession and $L_t^+ = n_t^+ + N_{t-1}^-$ workers in the boom.[9] Since both the long-run Markov probability of a boom and the long-run Markov probability of a recession are $\frac{1}{2}$, then average employment (over booms and recessions) is $L = \frac{1}{2}\left(L_t^+ + N_t^-\right)$.[10]

We assume that employing new workers involves a hiring cost h per worker. There are firing costs of two kinds. Let S be the firing costs that imply a transfer of income from the firm to the worker, e.g. severance payments or the requirement of advance notice. For simplicity, we call these costs S "severance payments". Let K be the firing costs that are not a transfer of income to the worker, e.g. trial costs. We assume that each incumbent worker's position is associated with firing costs $F = S + K$ per worker. The new recruits, in turn, have their positions associated with firing costs $f = s + k$ (the meaning of f, s and k is the same as that of the upper-case variables).

[8] We assume that on-the-job learning is costless.

[9] In the upturn, the number of employees in the firm is $L_t^+ = n_t^+ + N_{t-1}^-$. If the economy remains in a boom, the firm retains the same number of workers.

[10] The long-run Markov probabilities are calculated for $t \to \infty$. Thus, the level of employment in the first period of the firm (i.e. when the firm is created) becomes irrelevant.

We assume that all the wages are predetermined when the employment decisions are made. For clarity, we consider two different scenarios. (1.) In Section 3 the wages are exogenous, and all the workers have their positions associated with firing costs F (i.e. $F = f$). In this simple setting, we show the main reasons why on-the-job learning influences the employment effect of firing costs. (2.) In Section 4 we extend the analysis for endogenous wages, and we consider that firing new recruits is costless (i.e. $f = 0$). The distinction between severance payments (S) and the firing costs that are not a transfer of income to the worker (K) becomes relevant when wages are endogenous.

2.2 The hiring decision

In the hiring scenario, the firm's decision on employment is the solution of the following profit maximization:

$$\underset{n_t^+}{Max} Z^+ \left(n_t^+ + AN_{t-1}^-\right) - \frac{b}{2}\left(n_t^+ + AN_{t-1}^-\right)^2 - w_t^+ n_t^+ - W_t^+ N_{t-1}^- - hn_t^+ + \delta \Pi_{t+1}^e \tag{2}$$

where $n_t^+ + AN_{t-1}^-$ is employment in efficiency units of labor in the upturn, w_t^+ is the entrants' wage, W_t^+ is the incumbent workers' wage, Π_{t+1}^e is the expected value of discounted future profitability from $t+1$ forward, and $\delta = \frac{1}{1+r}$, where r is the discount factor. Solving (2), the marginal condition is:[11]

$$\begin{aligned} &\left[Z^+ - b\left(n_t^+ + AN_{t-1}^-\right)\right] - w_t^+ - \delta(1-P)f \\ &+\delta P \left\{ \frac{1}{1-\delta P}\left[A\left(Z^+ - bA\left(n_t^+ + N_{t-1}^-\right)\right) - W_{t+1}^+ - \delta(1-P)F\right] \right\} = h \end{aligned} \tag{3}$$

i.e. the expected marginal profitability must be equal to the cost of hiring the marginal worker h. This marginal condition may be described as follows: (i) in the current period t the new workers' marginal profitability is $\left[Z^+ - b\left(n_t + AN_{t-1}^-\right)\right] - w_t^+$. (ii) With probability $(1-P)$ the firm falls into a recession in period $t+1$ and fires some workers. The marginal new worker is fired and the firm pays the firing cost $-f$. (iii) With probability P the firm remains in the boom in period $t+1$ and retains all its workers. The

[11] See Appendix A for the derivation of the marginal conditions in (3) and (7).

expected value of the marginal worker's discounted profitability from $t+1$ forward is in curly brackets, where $A\left(Z^{+}-bA\left(n_t^{+}+N_{t-1}^{-}\right)\right)$ is the marginal product, and $\delta(1-P)F$ is the expected firing cost.

The expression in (3) determines the total number of people employed in the boom:

$$L_t^{+}=\frac{\left(Z^{+}-w_t^{+}-h\right)+\frac{\delta P}{1-\delta P}\left(AZ^{+}-W_{t+1}^{+}\right)-\delta(1-P)f-\frac{\delta^2 P(1-P)}{1-\delta P}F}{\left(1+\frac{\delta P}{1-\delta P}A^2\right)b}-\rho N_{t-1}^{-} \tag{4}$$

where $-\rho$ is the change in $L_t^{+}=n_t^{+}+N_{t-1}^{-}$ associated with a marginal increase in N_{t-1}^{-}. $-\rho$ equals:

$$-\rho=1+\frac{dn_t^{+}}{dN_{t-1}^{-}}=1-\frac{A+\frac{\delta P}{1-\delta P}A^2}{1+\frac{\delta P}{1-\delta P}A^2}<0 \tag{5}$$

Observe that when there is on-the-job learning, i.e. $A>1$, total employment in the upturn (L_t^{+}) responds negatively to increases in the number of incumbent workers in the firm (N_{t-1}^{-}). The reason is that incumbent workers are more productive than entrants, and thus one additional skilled worker in the firm displaces more than one new recruits in the next upturn (we can see in (5) that $-1>\frac{dn_t^{+}}{dN_{t-1}^{-}}\geq -A$). By contrast, without learning, i.e. $A=1$, an additional worker in the recession displaces one new recruit in the next upturn (i.e. $\frac{dn_t^{+}}{dN_{t-1}^{-}}=-1$), and total boom-time employment remains unchanged.

2.3 The firing decision

In the firing scenario, the firm's employment decision is the outcome of the following profit maximization:[12]

$$\underset{N_t^{-}}{Max}\ \ Z^{-}\left(AN_t^{-}\right)-\frac{b}{2}\left(AN_t^{-}\right)^2-W_t^{-}N_t^{-}-F\left(L_1^{+}-N_t^{-}\right)+\delta\Pi_{t+1}^{e} \tag{6}$$

[12]This is the employment decision in the first downturn the firm faces. For simplicity we consider that this occurs when the firm has spent at least two periods in a boom, so that workers have their positions associated with firing costs F. In any other downturn, the firm will just have the same number of workers as in this first downturn. Recall that in a recession all the workers are skilled workers.

where AN_t^- is employment in efficiency units in the recession and W_t^- is the wage. The marginal condition in the firing scenario is:

$$\begin{aligned} &A\left(Z^- - bAN_t^-\right) - W_t^- - \delta PF + \delta(1-P)\{h \\ &+(A-1)\left[Z^+ - b\left(n_{t+1}^+ + AN_t^-\right)\right] \\ &-\left(W_{t+1}^+ - w_{t+1}^+\right) - \delta(1-P)(F-f)\} = -F \end{aligned} \tag{7}$$

i.e. the profitability of the marginal skilled worker must be equal to the cost of firing him. This marginal condition may be explained as follows: (i) the skilled workers' marginal profitability in the current period is $A\left(Z^- - bAN_t^-\right) - W_t^-$. (ii) With probability P the recession continues and the expected profitability of the marginal worker equals $-F$. (iii) with probability $(1-P)$ economic conditions improve and the firm hires new workers. The expected value of the skilled workers' discounted marginal profitability from $t+1$ forward is in curly brackets: it equals the expected value of the unskilled workers' discounted marginal profitability from $t+1$ forward (which is equal to h, as equation (3) shows) plus the difference between the marginal product in $t+1$ of the skilled workers and the unskilled workers, $(A-1)\left[Z^+ - b\left(n_{t+1}^+ + AN_t^-\right)\right]$, the difference in the wage in $t+1$, $-\left(W_{t+1}^+ - w_{t+1}^+\right)$, and the difference in future firing costs, $-\delta(1-P)(F-f)$.[13]

The marginal condition (7) determines employment in the recession N_t^- for a given number of new recruits in a future upturn n_{t+1}^+. Using $L_{t+1}^+ = n_{t+1}^+ + N_t^-$ and solving for N_t^-, we obtain

$$\begin{aligned} N_t^- &= \frac{AZ^- - W_t^- + F - \delta PF}{\left[A^2 + \delta(1-P)(A-1)^2\right] b} \\ &+ \frac{\delta(1-P)\left[h + (A-1)Z^+ - \left(W_{t+1}^+ - w_{t+1}^+\right) - \delta(1-P)(F-f)\right]}{\left[A^2 + \delta(1-P)(A-1)^2\right] b} - \sigma L_{t+1}^+ \end{aligned} \tag{8}$$

where $-\sigma$ is the change in N_t^- associated with a marginal change in future employment L_{t+1}^+. $-\sigma$ equals:

$$-\sigma = -\frac{\delta(1-P)(A-1)}{A^2 + \delta(1-P)(A-1)^2} < 0 \tag{9}$$

[13] In an upturn in $t+1$, the only difference between the unskilled workers' expected marginal profitability from $t+1$ forward and that of the skilled workers is the marginal product and the wage in $t+1$, and the expected firing cost in $t+2$. This is so because after one period in the firm the unskilled workers become skilled workers.

These expressions show that, when $A > 1$, the number of employees in the recession (N_t^-) is inversely related to the number of employees in a future upturn (L_{t+1}^+): the lower is L_{t+1}^+, the higher is the expected profitability of the current marginal worker, and thereby the greater is the number of workers that the firm retains in the current recession. This is a relationship that does not exists when $A = 1$.

3 The employment effect of firing costs

This section considers the simple scenario in which wages are exogenous and all the workers have their positions associated with firing costs F (i.e. $F = f$). As noted, in this simple setting we can explain the main reasons why on-the-job learning influences the effect of firing costs on employment.

In this context, we can state the following proposition:

Proposition 1 *In the absence of on-the-job learning ($A = 1$), an increase in firing costs (F) has a positive effect on average employment (L) when $\delta < 1$.*

Under on-the-job learning ($A > 1$), the effect of F on L is weaker, and becomes negative when $A > A^*$, where $A^* = \frac{-a_2 - \sqrt{a_2^2 - 4a_1a_3}}{2a_1}$, where $a_1 = -\delta(1-P) - \delta^2(1-P)^2 + \delta^2 P(1-P)$; $a_2 = 3\delta^2(1-P)^2 - (1-\delta P)^2$; and $a_3 = 2(1-\delta P)^2 - 2\delta^2(1-P)^2$.

Proof. See Appendix B. ■

This proposition can be explained as follows.

In the absence of on-the-job learning, i.e. $A = 1$, the influence of a rise in firing costs on boom-time employment in (4) and the influence on recession-time employment in (8) are:

$$\frac{dL_t^+}{dF} = -\frac{\delta(1-P)}{b} < 0 \quad ; \quad \frac{dN_t^-}{dF} = \frac{(1-\delta P)}{b} > 0 \tag{10}$$

On the one hand, a rise in firing costs discourages hiring in the boom, since it increases the cost of firing a worker in the future. On the other hand, it discourages firing in the recession, since the firm has to pay the cost today. When $\delta < 1$, since firms that are engaged in firing must pay firing costs now whereas firms that are engaged in hiring may have to pay firing costs in the future, firing costs discourage firing more than hiring.[14] Thus a rise in firing costs has a positive effect on average employment $L = \frac{1}{2}\left(L_t^+ + N_t^-\right)$.

In the presence of on-the-job learning, i.e. $A > 1$, the effect of firing costs on boom-time employment in (4) is:

$$\frac{dL_t^+}{dF} = \left.\frac{dL_t^+}{dF}\right|_{\overline{N}^-} - \rho\frac{dN_{t-1}^-}{dF} \tag{11}$$

and on recession-time employment in (8) is

$$\frac{dN_t^-}{dF} = \left.\frac{dN_t^-}{dF}\right|_{\overline{L}^+} - \sigma\frac{dL_{t+1}^+}{dF} \tag{12}$$

These equations show that the influence of firing costs on short-run employment consists of two effects.

The first effect is the "direct effect" (represented by the first-right hand terms of both (11) and (12)), which is qualitatively the same as the one described in (10) for $A = 1$. This direct effect in the boom is negative and equals:

$$\left.\frac{dL_t^+}{dF}\right|_{\overline{N}^-} = -\frac{\delta\left(1-P\right)}{\left[1+\delta P(A^2-1)\right]b} < 0 \tag{13}$$

and, in the recession, it is positive and equals:

$$\left.\frac{dN_t^-}{dF}\right|_{\overline{L}^+} = \frac{1-\delta P}{\left[A^2+\delta(1-P)(A-1)^2\right]b} > 0 \tag{14}$$

The second effect is represented by the second right-hand terms of (11) and (12). To analyze this effect, equation (11) may be rewritten as[15]

$$\frac{dL_t^+}{dF} = \frac{1}{1-\rho\sigma}\left(\left.\frac{dL_t^+}{dF}\right|_{\overline{N}^-} - \rho\left.\frac{dN_t^-}{dF}\right|_{\overline{L}^+}\right) \tag{15}$$

and equation (12) as

$$\frac{dN_t^-}{dF} = \frac{1}{1-\rho\sigma}\left(\left.\frac{dN_t^-}{dF}\right|_{\overline{L}^+} - \sigma\left.\frac{dL_t^+}{dF}\right|_{\overline{N}^-}\right) \tag{16}$$

[14] See Bertola (1990).

[15] As noted, in the stationary equilibrium the firm has N_t^- workers in a recession and L_t^+ workers in a boom. Thus $L_t^+ = L_{t+1}^+$ and $N_t^- = N_{t-1}^-$. (11) and (12) is then a system of two equations with two unknowns.

This second effect exists because, in the presence of on-the-job learning, the hiring and firing decisions are interdependent. Recall that the number of workers that the firm needs in the boom depends on the number of workers that it has in a recession, as equation (4) shows. As noted, since incumbent workers are more productive than entrants, for each additional incumbent worker in the recession, in the upturn the firm will not hire a number of unskilled new workers greater than one. In other words, an additional skilled worker retained in the recession will be in the firm in the next upturn, and he will displace several (less productive) potential entrants. This means that the positive direct effect of firing costs in the recession translates into a negative effect in the boom, as equation (15) shows. We call this effect the "displacement effect in the boom", since former incumbent workers are displacing current entrants.

Similarly, the number of workers in a recession is conditioned by the number of new workers that the firm intends to hire in a possible future recovery, as equation (8) shows. Since firing costs reduce the number of new recruits, the firm has an incentive to retain additional workers in a recession. Thus, the negative direct effect of firing costs in the boom translates into a positive effect in the recession, as equation (16) shows. We call this effect the "replacement effect in the recession", since the firm is replacing future entrants with current incumbent workers.

In summary, in the boom both the "direct effect" and the "displacement effect" are negative, i.e. firing costs reduce boom-time employment. By contrast, in the recession both the "direct effect" and the "replacement effect" are positive, i.e. firing costs increase recession-time employment. In short, the existence of on-the-job learning does not change the result that turnover costs stabilize employment over the business cycle.

However, we can show that, in the presence of on-the-job learning, the effect of firing costs on average employment L is weaker and becomes negative when A is sufficiently large. By (15) and (16), the effect of firing costs on L equals:

$$\frac{dL}{dF} = \frac{1}{2(1-\rho\sigma)} \left[\left.\frac{dL_t^+}{dF}\right|_{\overline{N}^-} (1-\sigma) + \left.\frac{dN_t^-}{dF}\right|_{\overline{L}^+} (1-\rho) \right] \tag{17}$$

This effect is weaker when $A > 1$ than when $A = 1$ for two reasons.

The first reason is that $A > 1$ weakens the positive direct effect in the recession $\left.\frac{dN_t^-}{dF}\right|_{\overline{L}^+}$ more than it weakens the negative direct effect in the boom

$\left.\frac{dL_t^+}{dF}\right|_{\overline{N}^-}$, as we can see in equations (13) and (14). The explanation is the following. The firm is interested in the labor services in efficiency units. In a downturn, firing costs raise employment in efficiency units, and in an upturn, firing costs reduce employment in efficiency units. However, the number of workers that one efficiency unit represents is smaller in a recession than in a boom, because the workers that are fired (in a recession) are more productive than the workers that are hired (in a boom). As a consequence, firing costs increase average employment to a lesser extent and may even reduce it.

The second reason is related to the "displacement effect" and the "replacement effect". As noted, the fact that firing costs encourage firms to retain their skilled workers in a downturn has the side effect that, for each additional worker that the firm retains, it will not hire more than one unskilled workers in the upturn.[16] By the same token, the fact that firing costs discourage the hiring of new unskilled workers has the side effect that the firm will retain more skilled workers in the recession. The result is that the average productivity of the workforce is higher than it would otherwise be, and as a consequence, a smaller number of workers on average is required to produce a given cyclical trajectory of output. Thus firing costs increase average employment to a lesser extent and may even reduce it. We can see this in equation (17): the magnitude of ρ (i.e. the reduction in boom-time employment associated with an additional skilled worker in the previous recession, in (5)) is greater than the magnitude of σ (i.e. the increase in recession-time employment associated with one unskilled worker less in the future upturn, in (9)).

For these two reasons, on-the-job learning weakens the positive effect of firing costs on employment, and such an effect becomes negative when the amount of on-the-job learning is greater than a threshold value A^* (see Proposition 1).[17] For instance, $A^* = 1.08$ when $P = 0.5$, for a realistic value of $\delta = 0.9$.[18]

[16] Observe that the increase in recession-time employment due to firing costs may even translate into a reduction in average employment if the displacement of new entrants is sufficiently important, which occurs when $\rho > 1$.

[17] Observe in the equations above that in the particular case in which there is no discount factor (i.e. $\delta = 1$), firing costs have no effect on average employment in the absence of on-the-job learning. For the reasons explained above, the effect becomes negative for any $A > 1$.

[18] The values of A^* are equally plausible for other values of P: $A^* = 1.06$ when $P = 0.25$, and $A^* = 1.15$ when $P = 0.75$.

4 Firing costs and wage setting

This section considers endogenous wages. In this context, it is relevant the distinction between severance payments (S) and the firing costs that are not a transfer of income to the worker (K). Their effect on employment is analyzed in Sections 4.2 and 4.3, respectively.

4.1 Wage setting

We assume that only the skilled workers (the insiders) have firing costs (F) associated with their positions, whereas firing the unskilled workers (the entrants) is costless, i.e. $f = 0$. Thus the entrants have no market power and receive the reservation wage, i.e. the wage that makes them indifferent between employment and unemployment. The insiders, however, belong to a risk-neutral union and negotiate the wage with the firm before the employment decision is made, i.e. every time economic conditions change.

The insider wage is the solution of a Nash bargaining between the union and the firm. We assume that the union seeks to maximize the utility of the representative insider (the median voter). We also assume that, when firing, the firm follows an inverse seniority rule (last in, first-out), and that the representative insider is a "senior" insider who is not fired in the downturn. Accordingly, when setting the wage, the union does not care about the influence of a higher wage on employment.

In the Nash bargaining process, the firm's profit surplus at time t is the difference between the representative insider's expected profitability under agreement (X_t^i) and under disagreement (x_t^i).

Under agreement, the representative insider's expected profitability is:

$$X_t^i = AZ^i - \frac{b}{2} A^2 N_t^- - W_t^i + \theta_{t+1}^{ie}, \qquad i = +, - \tag{18}$$

where $AZ^i - \frac{b}{2}A^2 N_t^-$ is the insiders' average product in the current period, and θ_{t+1}^{ie} is the expected value of discounted future average profitability from $t+1$ forward.[19]

[19] Note that, in an upturn, the insiders' average product is $AZ^+ - \frac{b}{2}A^2 N_t^-$, since the negotiation occurs before the entrants are hired. In a recession, although the negotiation occurs before some workers are fired, the product of the representative insider is $AZ^- - \frac{b}{2}A^2 N_t^-$, since this insider remains in the firm in a downturn.

Under disagreement in the negotiation, we assume that workers go on strike for one period, and will try a new negotiation next period. We assume that the industrial action is costless to the workers but it has a cost α per capita for the firm. We also assume that the union can manipulate this cost α in accordance with its own interests. Thus the union will set α as high as possible, but not as high that the firm replaces the insiders with new workers. This implies that the insiders will remain in the firm both under agreement and under disagreement in the negotiation. Accordingly, we can assume that θ_{t+1}^{ie} is the same under agreement and under disagreement in the current negotiation. Thus, the representative insider's expected profitability under disagreement is:

$$x_t^i = \alpha + \theta_{t+1}^{ie}, \qquad i = +, - \tag{19}$$

By (18) and (19), the firm's profit surplus equals:

$$X_t^i - x_t^i = AZ^i - \frac{b}{2}A^2 N_t^- - W_t^i + \alpha, \qquad i = +, - \tag{20}$$

Regarding the worker's income surplus, it is the difference between the expected wage payments under agreement (Y_t^i) and under disagreement (y_t^i):

$$Y_t^i = W_t^i + \phi_{t+1}^{ie} \;\; ; \quad y_t^i = w^0 + \phi_{t+1}^{ie}, \qquad i = +, - \tag{21}$$

where w^0 is the worker's fall-back position, which can be interpreted as the financial support from family and friends, and ϕ_{t+1}^{ie} is the expected value of discounted future wages from $t+1$ forward. For the reasons explained above, ϕ_{t+1}^{ie} is the same under agreement and under disagreement. Thus the worker's income surplus equals:

$$Y_t^i - y_t^i = W_t^i - w^0, \qquad i = +, - \tag{22}$$

From (20) and (22), the Nash bargaining problem is:

$$\underset{W_t^i}{Max} \left(W_t^i - w^0\right)^{\mu} \left(Z^i A - \frac{b}{2}A^2 N_t^- - W_t^i + \alpha\right)^{1-\mu}, \qquad i = +, - \tag{23}$$

where μ is the union bargaining power. Solving, the insider wage is:[20]

$$W_t^i = (1-\mu)w^0 + \mu\left(AZ^i - \frac{b}{2}A^2 N_t^- - w^0 + \alpha\right), \qquad i = +, - \tag{24}$$

The union sets the cost of the strike α subject to the restriction that the representative insider is not replaced by a new worker. Thus, α must not be higher than the cost of firing the insider F plus the cost of hiring the new worker h minus the current profitability of the potential new worker ψ (which, for simplicity, we take as given).[21] Since the union seeks to maximize the wage in (24), then:

$$\alpha = (F+h) - \psi \tag{25}$$

Observe that the wage setting equation in (24) and (25) is quite standard in that the wage depends linearly on the profitability of labor and the firing cost. In this context, a rise in firing costs F has two effects on the insider wage:

$$\frac{dW_t^i}{dF} = \mu\left(1 - \frac{b}{2}A^2 \frac{dN_t^-}{dF}\right), \qquad i = +, - \tag{26}$$

On the one hand, it increases the cost that the firm bears under disagreement in the negotiation (α), and thus increases the negotiated wage. But, on the other hand, the bargained wage depends on the worker's average product: when a rise in firing costs increases the number of insiders in the firm N_t^-, it reduces the insiders' average product due to diminishing returns to labor, which in turn reduces the wage. It can be shown that the former effect dominates and thus a rise in firing costs increases the insider wage.[22]

[20] Although we can see in (8) that N_t^- depends on the insider wage, for simplicity we consider that the union takes the average product as given at the time of the negotiation. This simplifying assumption does not affect the qualitative results of the paper, as is shown in Appendix E.

[21] The actual restriction is that $\alpha - \theta_{t+1}^{ie} \leq F + h - \psi - \xi_{t+1}^{ie}$, where ξ_{t+1}^{ie} is the expected value of the current potential entrant's discounted profitability from $t+1$ forward. Nevertheless, when there is disagreement in the current negotiation, such a negotiation will have to take place again in the future, which implies that the current insiders will have to negotiate with the firm next period. Additionally, the current potential entrants would have to negotiate their wage with the firm as soon as they become insiders next period. This means that the current insiders and the potential entrants would be in a similar situation next period. Thus, for simplicity, we can assume $\theta_{t+1}^{ie} = \xi_{t+1}^{ie}$.

[22] The expression for $\frac{dN_t^-}{dF}$ is derived in the next sections for different scenarios.

Regarding the entrants' wage, since they receive their reservation wage (i.e. the wage that makes them indifferent between employment and unemployment), the present value of the entrant's expected income (including the future severance payments) must be equal to the one of an unemployed person:[23]

$$w_t^+ + \frac{\delta P}{1-\delta P}\left[W_{t+1}^+ + \delta(1-P)S\right] = \frac{B}{1-\delta P} \tag{27}$$

where B is the income per period that an unemployed person receives, which we assume constant. Thus the contract between the firm and the entrant is designed to compensate any increase in the entrant's expected income in the future by reducing the entrant's current wage, so that the present value of the firm's stream of payments to the worker remains unchanged.

4.2 The employment effect of severance payments

The following proposition summarizes the effect of severance payments on employment:

Proposition 2 *In the absence of on-the-job learning ($A = 1$), an increase in severance payments (S) has a positive effect on average employment (L), when $\delta < 1$.*

Under on-the-job learning ($A > 1$), this positive effect of S on L is weaker, and becomes negative when A is sufficiently large so that $\rho > 1$.

Proof. *See Appendix C.* ■

This proposition can be explained as follows.[24]

In the absence of on-the-job learning, the influence of severance payments on N_t^- and L_t^+ are:

$$\left(\frac{dN_t^-}{dS}\right)_{no\ ojl} = \frac{(1-\delta P) - \frac{\delta^2(1-P)^2}{1-\delta P} - \mu\left(1+\frac{\delta(1-P)}{1-\delta P}\right)}{b\left[1-\frac{\mu}{2}\left(1+\frac{\delta(1-P)}{1-\delta P}\right)\right]}; \qquad \left(\frac{dL_t^+}{dS}\right)_{no\ ojl} = 0 \tag{28}$$

[23]Note that W_t^- does not appear in the expression because current entrants are fired in a future recession.

[24]The expressions in this section are derived in the proof of Proposition 2.

As (28) shows, a rise in severance payments has no influence on the firm's employment decision in the upturn -a conclusion already reached by Lazear (1990). The reason is that the present value of the firm's stream of payments to the entrants is unchanged when entrants receive their reservation wage, and thus a rise in expected severance payments in the future is offset by a lower entrant's wage.

In the recession, by contrast, severance payments affect employment. This effect is ignored in Lazear (1990), but it is crucial to determine the influence of severance payments (S) on average employment. As (28) shows, severance payments have a direct positive effect on recession-time employment and an indirect negative effect via the negotiated wage (represented by the terms with μ). Here we consider the relevant case in which the direct positive effect dominates, i.e. $\left(\frac{dN_t^-}{dS}\right)_{no\ ojl} > 0$. Thus, in the absence of on-the-job learning, severance payments increase average employment because they encourage the retention of incumbent workers in a downturn (when the firm is engaged in firing).

In the presence of on-the-job learning, the influence of S on N_t^- and L_t^+ are:

$$\left(\frac{dN_t^-}{dS}\right)_{ojl} = \frac{(1-\delta P) - \frac{\delta^2(1-P)^2}{1-\delta P} - \mu\left(1+\frac{\delta(1-P)}{1-\delta P}\right)}{\left\{A^2\left[1-\frac{\mu}{2}\left(1+\frac{\delta(1-P)}{1-\delta P}\right)\right] + \delta(1-P)(A-1)^2\right\} b};$$
$$\left(\frac{dL_t^+}{dS}\right)_{ojl} = -\rho\left(\frac{dN_t^-}{dS}\right)_{ojl} \tag{29}$$

As we can see, the positive effect of severance payments on recession-time employment (N_t^-) is weaker when $A > 1$ than when $A = 1$, and the reason is the same as in Section 3: one efficiency unit of labor in the downturn represents less workers (since the incumbent workers are more productive).

Additionally, severance payments also have an "displacement effect" on boom-time employment. Recall that when $A > 1$, i.e. when insiders are more productive than entrants, each additional insider that the firm retains in the recession displaces several potential entrants in a future upturn. This implies that the positive influence of severance payments on recession-time employment translates into a negative influence on boom-time employment.

By (29), we can write the effect of S on average employment as:

$$\left(\frac{dL}{dS}\right)_{ojl} = \frac{1}{2}(1-\rho)\left(\frac{dN_t^-}{dS}\right)_{ojl} \tag{30}$$

The positive effect of severance payments on average employment is weaker when $A > 1$ than when $A = 1$, both because the magnitude of $\left(\frac{dN_t^-}{dS}\right)_{ojl}$ is smaller and because $\rho > 0$. When A is sufficiently large so that $\rho > 1$, severance payments reduce average employment.

4.3 The employment effect of firing costs that are not a transfer of income to the worker

We can summarize the effect of these firing costs represented by K in the following proposition:

Proposition 3 *1. When* $\mu < \widehat{\mu} = \frac{-\delta^2 P(1-P)+(1-\delta P)-\delta^2(1-P)^2}{\left(1+\frac{\delta(1-P)}{1-\delta P}\right)\left(1-\frac{\delta^2 P(1-P)}{2}\right)}$*: a) in the absence of on-the-job learning* ($A = 1$)*, a rise in firing costs* (K) *increases average employment* (L)*; b) under on-the-job learning* ($A > 1$)*, this positive effect of* K *on* L *is weaker, and becomes negative when* $A > \widehat{A}$ *(where* $\widehat{A}$ *depends on* μ*,* P *and* δ*).*

2. When $\mu > \widehat{\mu}$*: a) when* $A = 1$*, a rise in* K *reduces average employment* (L)*; b) when* $\mu < \widehat{\widehat{\mu}}$ *(where* $\widehat{\widehat{\mu}}$ *depends on* A*,* P *and* δ*), the negative effect of* K *on* L *is weaker when* $A = 1$ *than when* $A > 1$*; c) when* $\mu > \widehat{\widehat{\mu}}$*, the negative effect of* K *on* L *is weaker when* $A > 1$ *than when* $A = 1$*.*

Proof. See Appendix D. ■

This proposition can be explained as follows.

1. When the bargaining power of the union is not too high, i.e. $\mu < \widehat{\mu}$, a rise in firing costs K increases average employment in the absence of on-the-job learning. On the one hand, since here firing costs are not a payment that the worker receives when fired, the entrant's reservation wage does not fall when the future firing cost rises. As a consequence, a rise in this type of firing costs discourages hiring.[25] On the other hand, although firing costs

[25] The entrant wage is however offsetting any increase in future insider wages, which implies that firing costs have no influence on boom-time employment via the insider wage. Thus, by (4), the negative employment effect of firing costs in the boom only consists of the direct effect.

have a negative effect on recession-time employment via the insider wage, this effect is not very important since the power of the union is not too high.[26] The effect of firing costs in the recession is positive and still greater than the negative effect in the boom, and thus firing costs increase average employment. The role that on-the-job learning plays here is the same as the one described in Section 3 (Proposition 1), i.e. the positive effect of firing costs on L is weaker in the presence of on-the-job learning, and in fact becomes negative when the amount of on-the-job learning is sufficiently large so that $A > \widehat{A}$. For instance, when $\delta = 0.9$, $P = 0.5$, and $\mu = 0.05$, the effect of a rise in K on L is positive for $A = 1$, becomes zero for $\widehat{A} = 1.25$, and is negative for any $A > 1.25$.

2. These qualitative results also hold when $\mu > \widehat{\mu}$ but μ is not too high. For instance, when $\mu = 0.1$, the effect of K on L is negative for $A = 1$, and becomes more contractionary for $A > 1$. However, the opposite may occur when μ is sufficiently large so that $\mu > \widehat{\widehat{\mu}}$:[27] on-the-job learning can weaken the negative effect of K on L. To explain the intuition, we use the following example. Consider the extreme case in which μ is so large (i.e. a rise in firing costs increases the insider wage so much) that the positive direct effect of K on N_t^- is offset by the negative effect via the higher insider wage. This implies that K only has a direct effect on the hiring decision, which is negative. Thus, a rise in K reduces average employment (L). In this case, the existence of A weakens this reduction. The main reason is that firing costs discourage hiring to a lesser extent because the firm expects the new workers to learn on the job and be more productive in the future.

5 Conclusions

This paper has shown how on-the-job learning influences the way in which firing costs affect employment. The reason why on-the-job learning plays this role is that it creates a productivity differential between incumbent workers and new recruits, and thereby influences how many new recruits are necessary to replace a given number of incumbents who have been fired in the previous recession. Thus, the firms' hiring and firing decisions become interdependent.

On this account, the number of workers the firm retains in a recession

[26] In the recession, as in the case of severance payments, firing costs (K) influence employment both directly and via the insider wage.

[27] See the proof of Proposition 3.

affects the number of workers it needs to hire in a subsequent recovery. Firing costs encourage firms to retain more workers in a recession. The greater the amount of on-the-job learning, the greater the productivity differential between incumbents and new recruits, and thus the fewer new recruits need to be hired in the recovery. In this way, on-the-job learning imparts a contractionary influence on the employment repercussions of firing costs.

This influence is strengthened because firing costs increase employment (in efficiency units) in a recession and reduce employment (in efficiency units) in a boom, but the number of workers that these efficiency units represent is smaller in a recession than a boom. On-the-job learning generates this channel whereby firing costs may reduce average employment over the business cycle. As noted, these results imply that skill-biased technological change, falling on firm-specific skills, can make firing costs less beneficial for employment.

Only when a rise in firing costs increases the insider wage so much that such a rise reduces average employment, it may occur that the existence of on-the-job learning weakens this reduction.

A Appendix. Derivation of (3) and (7)

In the boom, the solution of (2) is:

$$\left[Z^+ - b\left(n_t^+ + AN_{t-1}^-\right)\right] - w_t^+ - \delta(1-P)f + \delta P\Pi_{t+1}^{++\prime} = h \tag{31}$$

where $\Pi_{t+1}^{++\prime}$ is the expected value of discounted marginal profitability from $t+1$ forward if the firm remains in the boom in period $t+1$. $\Pi_{t+1}^{++\prime}$ equals

$$\Pi_{t+1}^{++\prime} = A\left[Z^+ - bA\left(n_t^+ + N_{t-1}^-\right)\right] - W_{t+1}^+ + \delta P\Pi_{t+2}^{++\prime} - \delta(1-P)F \tag{32}$$

Since Z_t is a stationary process, it holds that $\Pi_{t+1}^{++\prime} = \Pi_{t+2}^{++\prime}$, and we have that:

$$\Pi_{t+1}^{++\prime} = \frac{1}{1-\delta P}\left\{A\left[Z^+ - bA\left(n_t^+ + N_{t-1}^-\right)\right] - W_{t+1}^+ - \delta(1-P)F\right\} \tag{33}$$

Substituting equation (33) into (31), the marginal condition in the hiring scenario is the expression in (3).

In the recession, the solution of (6) is:

$$A\left(Z^- - bAN_t^-\right) - W_t^- - \delta PF + \delta(1-P)\Pi_{t+1}^{+\prime} = -F \tag{34}$$

where $\Pi_{t+1}^{+\prime}$ is the expected value of the skilled workers' discounted marginal profitability from $t+1$ forward when there is an upturn in $t+1$. $\Pi_{t+1}^{+\prime}$ equals:

$$\Pi_{t+1}^{+\prime} = A\left[Z^{+} - b\left(n_{t+1}^{+} + AN_{t}^{-}\right)\right] - W_{t+1}^{+} - \delta(1-P)F + \delta P \Pi_{t+2}^{++\prime} \qquad (35)$$

Using equation (31), we can rewrite equation (35) as:

$$\Pi_{t+1}^{+\prime} = h + (A-1)\left[Z^{+} - b\left(n_{t+1} + AN_{t}^{-}\right)\right] - (W_{t+1}^{+} - w_{t+1}^{+}) - \delta(1-P)(F-f) \qquad (36)$$

Substituting (36) into (34), we obtain the marginal condition in (7).

B Appendix. Proof of Proposition 1

By (17), (13) and (14), $\frac{dL}{dF} > 0$ is weaker when $A > 1$ than when $A = 1$ when the following condition is fulfilled:

$$\begin{aligned} & -\frac{\delta(1-P)}{X(1-\rho\sigma)}\left(1 - \frac{\delta(1-P)(A-1)}{Y}\right) \\ & +\frac{1-\delta P}{Y(1-\rho\sigma)}\left(1 - \frac{(1-\delta P)(A-1)}{X}\right) \\ < \quad & -\frac{\delta(1-P)}{1} + \frac{1-\delta P}{1} \end{aligned} \qquad (37)$$

where $X = 1 + \delta P(A^2 - 1)$ and $Y = A^2 + \delta(1-P)(A-1)^2$. (37) can be rewritten as:

$$\delta(1-P)\left[1 - \frac{Y - \delta(1-P)(A-1)}{YX(1-\rho\sigma)}\right] - (1-\delta P)\left(1 - \frac{X - (1-\delta P)(A-1)}{YX(1-\rho\sigma)}\right) < 0 \qquad (38)$$

which simplifies to:

$$\frac{1-\delta P}{\delta(1-P)} > \frac{YX(1-\rho\sigma) + \delta(1-P)(A-1) - Y}{YX(1-\rho\sigma) + (1-\delta P)(A-1) - X} \qquad (39)$$

This inequality holds for any values of the parameters since $X < Y$.

The threshold value A^* from which $\frac{dL}{dF}$ becomes negative is the solution of the equation $A^2 a_1 + A a_2 + a_3 = 0$, where $a_1 = -\delta(1-P) - \delta^2(1-P)^2 + \delta^2 P(1-P)$; $a_2 = 3\delta^2(1-P)^2 - (1-\delta P)^2$; and $a_3 = 2(1-\delta P)^2 - 2\delta^2(1-P)^2$. This equation $A^2 a_1 + A a_2 + a_3 = 0$ has only one positive solution since $a_1 < 0$ and $a_3 > 0$, and this solution is $A^* = \frac{-a_2 - \sqrt{a_2^2 - 4a_1 a_3}}{2a_1}$.

C Appendix. Proof of Proposition 2

When $A = 1$, by (8), (26) and (27), the effect of S on N_t^- equals:

$$\left(\frac{dN_t^-}{dS}\right)_{no\ ojl} = \frac{(1-\delta P) - \frac{\delta^2(1-P)^2}{1-\delta P} - \left(\frac{dW_t^-}{dS} + \frac{\delta(1-P)}{1-\delta P}\frac{dW_{t+1}^+}{dS}\right)}{b} \tag{40}$$

Substituting (26) into (40), we obtain $\left(\frac{dN_t^-}{dS}\right)_{no\ ojl}$ in (28). By (4) and (27), we obtain $\left(\frac{dL_t^+}{dS}\right)_{no\ ojl}$ in (28). As noted in Section 4.2, we consider the relevant case in which $\left(\frac{dN_t^-}{dS}\right)_{no\ ojl} > 0$. Thus $\left(\frac{dL}{dS}\right)_{no\ ojl} > 0$.

Similarly, when $A > 1$, by (8), (26) and (27), we obtain $\left(\frac{dN_t^-}{dS}\right)_{ojl}$ in (29), and by (4) and (27), we obtain $\left(\frac{dL_t^+}{dS}\right)_{ojl}$ in (29). The effect of S on L is in (30). For $A \in (1, A^*)$, $\left(\frac{dL}{dS}\right)_{ojl}$ is positive and weaker than $\left(\frac{dL}{dS}\right)_{no\ ojl}$. Figure 1 illustrates it for $\delta = 0.9$ and $\mu = 0.05$, and for the following values of P: 0, 0.3, 0.5, 0.7, 0.9. The qualitative result is the same for any other plausible parameter values. $\left(\frac{dL}{dS}\right)_{ojl}$ becomes negative when ρ in (5) is smaller than 1, i.e. when $A > A^*$, where $A^* = \frac{(1-\delta P)-\sqrt{(1-\delta P)^2-8\delta P(1-\delta P)}}{2\delta P}$. Note that A^* only exists when $\delta P < \frac{1}{9}$.[28]

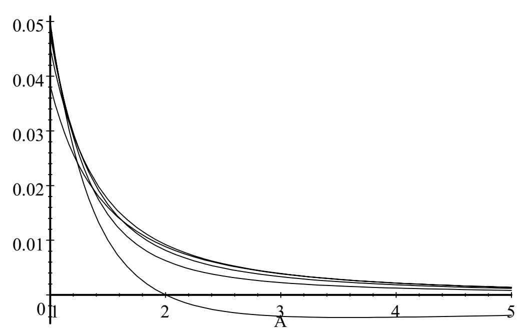

Figure 1

[28]Observe that $\left(\frac{dL}{dS}\right)_{ojl}$ may become positive again when $A > A^{**} = \frac{(1-\delta P)+\sqrt{(1-\delta P)^2-8\delta P(1-\delta P)}}{2\delta P}$, but this value is implausibly high for the relevant values of the parameters. For instance, $A^* = 3$ and $A^{**} = 6$ when $\delta P = \frac{1}{10}$; and $A^* = 2.2$ and $A^{**} = 16.7$ when $\delta P = \frac{1}{20}$. As noted, A^* and A^{**} do not exist when $\delta P \geq \frac{1}{9}$.

D Appendix. Proof of Proposition 3

1. When $\left(\frac{dL}{dK}\right)_{no\ ojl} > 0$, this effect becomes weaker when $A > 1$, i.e. $\left(\frac{dL}{dK}\right)_{no\ ojl} > \left(\frac{dL}{dK}\right)_{ojl}$.

Proof: By (4), (8), (27) and (26), when $A = 1$:[29]

$$\left(\frac{dL_t^+}{dK}\right)_{no\ ojl} = -\frac{\delta^2 P(1-P)}{b} < 0 \tag{41}$$

$$\left(\frac{dN_t^-}{dK}\right)_{no\ ojl} = \frac{(1-\delta P) - \delta^2(1-P)^2 - \mu\left(1 + \frac{\delta(1-P)}{1-\delta P}\right)}{b\left[1 - \frac{\mu}{2}\left(1 + \frac{\delta(1-P)}{1-\delta P}\right)\right]} \tag{42}$$

By (41) and (42), $\left(\frac{dL}{dK}\right)_{no\ ojl}$ equals:

$$\left(\frac{dL}{dK}\right)_{no\ ojl} = \left(-x + \frac{y}{z}\right)\frac{1}{2b} \tag{43}$$

where $x = \delta^2 P(1-P)$, $y = (1-\delta P) - \delta^2(1-P)^2 - \mu\left(1 + \frac{\delta(1-P)}{1-\delta P}\right)$ and $z = 1 - \frac{\mu}{2}\left(1 + \frac{\delta(1-P)}{1-\delta P}\right)$. Since $\left(\frac{dL}{dK}\right)_{no\ ojl} > 0$, then $x < \frac{y}{z}$.

By (4), (8), (27) and (26), when $A > 1$:

$$\left(\frac{dL_t^+}{dK}\right)_{ojl} = -\frac{\delta^2 P(1-P)}{\left[1 + \delta P(A^2-1)\right]b} - \rho\frac{dN_{t-1}^-}{dK} \tag{44}$$

$$\left(\frac{dN_t^-}{dK}\right)_{ojl} = \frac{(1-\delta P) - \delta^2(1-P)^2 - \mu\left(1 + \frac{\delta(1-P)}{1-\delta P}\right)}{\left\{A^2\left[1 - \frac{\mu}{2}\left(1 + \frac{\delta(1-P)}{1-\delta P}\right)\right] + \delta(1-P)(A-1)^2\right\}b} - \widetilde{\sigma}\frac{dL_{t+1}^+}{dK} \tag{45}$$

where

$$\widetilde{\sigma} = \frac{\delta(1-P)(A-1)}{A^2\left[1 - \frac{\mu}{2}\left(1 + \frac{\delta(1-P)}{1-\delta P}\right)\right] + \delta(1-P)(A-1)^2} \tag{46}$$

[29] This expression is similar to (13), although now for firing costs to have a direct influence on the hiring decision, the marginal worker must first become an insider, which occurs with probability P.

By (44) and (45), $\left(\frac{dL}{dK}\right)_{ojl}$ equals:

$$\left(\frac{dL}{dK}\right)_{ojl} = \frac{1}{2b\,(1-\rho\widetilde{\sigma})}\left[-\frac{x}{1+\delta P(A^2-1)}(1-\widetilde{\sigma}) + \frac{y}{A^2 z+\delta(1-P)(A-1)^2}(1-\rho)\right] \tag{47}$$

where $\widetilde{\sigma}$ and ρ are in equations (46) and (5), respectively. By (43) and (47):

$$\left(\frac{dL}{dK}\right)_{no\ ojl} - \left(\frac{dL}{dK}\right)_{ojl} = \tag{48}$$
$$-x\left(1-\frac{1-\widetilde{\sigma}}{1+\delta P(A^2-1)}\frac{1}{1-\rho\widetilde{\sigma}}\right)+\frac{y}{z}\left(1-z\frac{(1-\rho)}{A^2 z+\delta(1-P)(A-1)^2}\frac{1}{1-\rho\widetilde{\sigma}}\right)$$

By (46) and (5), we know that $\frac{1-\widetilde{\sigma}}{1+\delta P(A^2-1)} > \frac{1-\rho}{A^2 z+\delta(1-P)(A-1)^2}$. Thus the expression in (48) is positive when $x < \frac{y}{z}$. Then it holds that $\left(\frac{dL}{dK}\right)_{no\ ojl} > \left(\frac{dL}{dK}\right)_{ojl}$.

2. When $\left(\frac{dL}{dK}\right)_{no\ ojl} > 0$, this positive effect may become negative when $A > \widehat{A}$.

Proof: By (47), $\left(\frac{dL}{dK}\right)_{ojl} < 0$ when

$$a_1 A^2 + a_2 A + a_3 < 0 \tag{49}$$

where $a_1 = \left[\delta P\frac{y}{x} - z - \delta(1-P)\right]$, $a_2 = \left[-\frac{y}{x}(1-\delta P) + 3\delta(1-P)\right]$ and $a_3 = \left[2(1-\delta P)\frac{y}{x} - 2\delta(1-P)\right]$. We can show that there is a unique $\widehat{A}$ such that for any $A > \widehat{A}$, (49) holds. $\widehat{A}$ is the solution of $a_1A^2+a_2A+a_3 = 0$. Note that a_2 and a_3 cannot be both negative.[30] When a_1 and a_3 have opposite sign, there is a unique positive solution for $\widehat{A}$. When a_1 and a_3 have the same sign, there are three possibilities: a) When $a_1 < 0$, $a_2 > 0$ and $a_3 < 0$, there is a unique solution $\widehat{A} > 1$;[31] b) When $a_1 > 0$, $a_2 > 0$ and $a_3 > 0$, there

[30] a_2 is negative when $\frac{y}{x} > \frac{3\delta(1-P)}{(1-\delta P)}$ and a_3 is negative when $\frac{y}{x} < \frac{\delta(1-P)}{(1-\delta P)}$, which is not compatible.

[31] When $a_1 < 0$, $a_2 > 0$ and $a_3 < 0$, the equation $a_1A^2 + a_2A + a_3 = 0$ has two positive solutions. We show that the smallest solution is smaller than 1, so that it is irrelevant, while the largest solution is greater than 1. The smallest solution is $\widehat{A}_{1s} = \frac{-a_2+\sqrt{a_2^2-4a_1a_3}}{2a_1}$, which

is no solution;[32] and c) When $a_1 > 0$, $a_2 < 0$ and $a_3 > 0$, there is also no solution.[33] In both case c) and b) what happens is that $\left(\frac{dL}{dK}\right)_{no\ ojl}$ is positive for any A, although it still holds that $\left(\frac{dL}{dK}\right)_{no\ ojl} > \left(\frac{dL}{dK}\right)_{ojl}$(the result proved above).

3. When $\left(\frac{dL}{dK}\right)_{no\ ojl} < 0$, it holds that, for a specific $\overline{A} > 1$: a) when $\mu < \widehat{\widehat{\mu}}$, $\left(\frac{dL}{dK}\right)_{no\ ojl} > \left(\frac{dL}{dK}\right)_{\overline{A}>1}$; b) when $\mu > \widehat{\widehat{\mu}}$, $\left(\frac{dL}{dK}\right)_{no\ ojl} < \left(\frac{dL}{dK}\right)_{\overline{A}>1}$.

Proof: We can see in (48) that the greater is μ, the smaller is $\frac{y}{z}$, and the smaller is $\frac{1-\widetilde{\sigma}}{1+\delta P(A^2-1)}$ relative to $\frac{1-\rho}{A^2 z+\delta(1-P)(A-1)^2}$, and thus the more likely (48) is negative. In fact, there is a threshold value $\widehat{\widehat{\mu}}$ for which (48) is zero. The value of $\widehat{\widehat{\mu}}$ depends on the specific $\overline{A}$ that we are considering. In the table below, we consider several values of A ($A_1 = 1.1$, $A_2 = 2$ and $A_3 = 3$) and calculate the $\widehat{\widehat{\mu}}$ for two relevant values of the discount factor ($\delta_1 = 0.9$ and $\delta_2 = 0.5$) and three representative values of P ($P_1 = 0.25$, $P_2 = 0.5$ and $P_3 = 0.75$):

	For	$\delta_1 = 0.9$	
	$A_1 = 1.1$	$A_2 = 2$	$A_3 = 3$
$P_1 = 0.25$	0.14	0.13	0.12
$P_2 = 0.5$	0.13	0.11	0.10
$P_3 = 0.75$	0.10	0.09	0.08

	For	$\delta_2 = 0.5$	
	$A_1 = 1.1$	$A_2 = 2$	$A_3 = 3$
$P = 0.25$	0.508	0.506	0.503
$P = 0.5$	0.505	0.499	0.493
$P = 0.75$	0.497	0.489	0.485

is smaller than 1 when $a_2 - \sqrt{a_2^2 - 4a_1a_3} < -2a_1$. Rearranging, this expression equals $a_2 + 2a_1 < \sqrt{a_2^2 - 4a_1a_3}$, which in turn equals $4a_1(a_1 + a_2 + a_3) < 0$. This inequality is satisfied, since we are in the case in which for $A = 1$, the expression in (49) is positive, i.e. $(a_1 + a_2 + a_3) > 0$. The largest solution is $\widehat{A}_{1l} = \frac{a_2+\sqrt{a_2^2-4a_1a_3}}{-2a_1}$. It is > 1 when $a_2 + 2a_1 > -\sqrt{a_2^2 - 4a_1a_3}$. This inequality holds a) when $a_2 + 2a_1 > 0$, and b) when $a_2 + 2a_1 < 0$ (in this latter case, it holds that $(a_2 + 2a_1)^2 < a_2^2 - 4a_1a_3$, which equals $4a_1(a_1 + a_2 + a_3) < 0$. This inequality is satisfied, since $a_1 < 0$ and $(a_1 + a_2 + a_3) > 0$).

[32] In this case the two solutions for $\widehat{A}$ are negative.

[33] We define $\widehat{a}_1 = \left[\delta P\frac{y}{x} - \frac{y}{x} - \delta(1-P)\right] < a_1$. We can prove that $a_2^2 - 4\widehat{a}_1a_3 < 0$ when $\widehat{a}_1 > 0$ and $a_3 > 0$, which would imply that $a_2^2 - 4a_1a_2 < 0$, and thus that there is no solution for $\widehat{A}$. If we call $\eta_1 = \frac{x}{y}(1-\delta P)$ and $\eta_2 = \delta(1-P)$, then $\widehat{a}_1 = \eta_1 - \eta_2 > 0$, $a_2 = -\eta_1 + 3\eta_2 < 0$ and $a_3 = 2(\eta_1 - \eta_2) > 0$. The expression $a_2^2 - 4\widehat{a}_1a_2$ is then equal to $2\eta_1[-2(\eta_1 - \eta_2) + (-\eta_1 + 3\eta_2)] - (\eta_1^2 - \eta_2^2)$, which is smaller than 0.

In the particular case in which the first right-hand term of (45) is zero, K only affects directly the firm's hiring decision and, thus:

$$\frac{dL}{dK} = -\frac{\delta^2 P(1-P)}{2\left[1+\delta P(A^2-1)\right]b}(1-\widetilde{\sigma}) < 0 \qquad (50)$$

When $A > 1$ this effect is less negative because $-\frac{\delta^2 P(1-P)}{2[1+\delta P(A^2-1)]b}$ is weaker and $\widetilde{\sigma} > 0$.

E Appendix. The wage

The expression for the wage W_t^i is the solution of (23):

$$W_t^i = (1-\frac{\mu}{\phi})\omega^0 + \frac{\mu}{\phi}\left(AZ^i - \frac{b}{2}A^2 N_t^- + \alpha\right), \qquad i = +,- \qquad (51)$$

where $\phi = 1 + (1-\mu)\frac{b}{2}A^2\frac{dN_t^-}{dW_t^i}$. (1.) In the case of the insider wage in the boom W_t^+, the term $\frac{\partial N_t^-}{\partial W_t^+}$ equals zero, since when W_t^+ is being negotiated, N_t^- is already given. Thus, in that case $\phi = 1$ and the expression for the wage is (24). (2.) In the recession, the term ϕ appears because, since the wage negotiation occurs before the employment decision N_t^-, the union can take into account that a rise in the insider wage reduces N_t^- and increases the average product. (The expression for $\frac{dN_t^-}{dW_t^-} = \frac{-1}{(1-\rho\sigma)[A^2+(1-P)(A-1)^2]b}$ is obtained from (8) and (4).) We can show that including the term ϕ in the analysis does not change the qualitative results:

(A.) In the analysis of the employment effect of severance payments (S) of Section 4.2, the expression $\left(\frac{dN_t^-}{dS}\right)_{ojl}$ becomes:

$$\left(\frac{dN_t^-}{dS}\right)_{ojl} = \frac{(1-\delta P) - \frac{\delta^2(1-P)^2}{1-\delta P} - \mu\left(\frac{1}{\phi} + \frac{\delta(1-P)}{1-\delta P}\right)}{\left\{A^2\left[1 - \frac{\mu}{2}\left(\frac{1}{\phi} + \frac{\delta(1-P)}{1-\delta P}\right)\right] + \delta(1-P)(A-1)^2\right\} b} \qquad (52)$$

which is smaller when $A > 1$ than when $A = 1$.

(B.) In the analysis of the employment effect of K of Section 4.3, the expression $\left(\frac{dN_t^-}{dK}\right)_{ojl}$ becomes:

$$\frac{dN_t^-}{dK} = \frac{(1-\delta P) - \delta^2(1-P)^2 - \mu\left(\frac{1}{\phi} + \frac{\delta(1-P)}{1-\delta P}\right)}{\left\{A^2\left[1 - \frac{\mu}{2}\left(\frac{1}{\phi} + \frac{\delta(1-P)}{1-\delta P}\right)\right] + \delta(1-P)(A-1)^2\right\} b} - \widetilde{\sigma}\frac{dL_{t+1}^+}{dK} \quad (53)$$

where

$$\widetilde{\sigma} = \frac{\delta(1-P)(A-1)}{A^2\left[1 - \frac{\mu}{2}\left(\frac{1}{\phi} + \frac{\delta(1-P)}{1-\delta P}\right)\right] + \delta(1-P)(A-1)^2} \quad (54)$$

The proof of Proposition 3 presented in Appendix D also holds using these two expressions, where now $x = \delta^2 P(1-P)$, $y = (1-\delta P) - \delta^2(1-P)^2 - \mu\left(\frac{1}{\phi} + \frac{\delta(1-P)}{1-\delta P}\right)$ and $z = 1 - \frac{\mu}{2}\left(\frac{1}{\phi} + \frac{\delta(1-P)}{1-\delta P}\right)$.

F Appendix. Nonlinear marginal product function

Let $\mathcal{F}^i(\cdot)$ be the production function, $i = +, -$. $\mathcal{F}^{+\prime}(n_t^+ + AN_{t-1}^-)$ is the marginal product in an upturn, $A\mathcal{F}^{+\prime}\left(An_t^+ + AN_{t-1}^-\right)$ is the marginal product if the boom persists, and $A\mathcal{F}^{-\prime}(AN_t^-)$ is the marginal product in a recession. The effect of F on L_t^+ and on N_t^- are in equations (11) and (12), where now $\rho = \frac{(1-P)(-\mathcal{F}^{+\prime\prime}(n_t^+ + AN_{t-1}^-))(A-1)}{\eta}$, $\sigma = \frac{(1-P)(-\mathcal{F}^{+\prime\prime}(n_t^+ + AN_{t-1}^-))(A-1)}{\zeta}$, $\left.\frac{dL_t^+}{dF}\right|_{\overline{N}^-} = -\frac{1-P}{\zeta}$ and $\left.\frac{dN_t^-}{dF}\right|_{\overline{L}^+} = \frac{1-P}{\eta}$, where $\eta = -A^2\mathcal{F}^{-\prime\prime}(AN_t^-) - (1-P)(A-1)^2\mathcal{F}^{+\prime\prime}(n_{t+1}^+ + AN_t^-)$ and $\zeta = -\mathcal{F}^{+\prime\prime}(n_t^+ + AN_{t-1}^-) - P(A^2-1)\mathcal{F}^{+\prime\prime}(An_t^+ + AN_{t-1}^-)$.

When $\mathcal{F}^{i\prime\prime}(\cdot)$ is a constant, it holds that $(\eta - \zeta)_{A>1} > (\eta - \zeta)_{A=1}$. This is the case studied in Proposition 1.

When the marginal product function is nonlinear and so $\mathcal{F}^{i\prime\prime}(\cdot)$ depends on employment in efficiency units, the rise in A also affects $\mathcal{F}^{i\prime\prime}(\cdot)$ since it affects employment in efficiency units. In this case, it occurs that $(\eta - \zeta)_{A>1} > (\eta - \zeta)_{A=1}$ when:

$$\begin{aligned} & -A^2\mathcal{F}^{-\prime\prime}(AN_t^-) - (1-P)(A-1)^2\mathcal{F}^{+\prime\prime}(n_{t+1}^+ + AN_t^-) - \\ & \left(-\mathcal{F}^{+\prime\prime}(n_t^+ + AN_{t-1}^-) - P(A^2-1)\mathcal{F}^{+\prime\prime}(An_t^+ + AN_{t-1}^-)\right) > \\ & -\mathcal{F}^{-\prime\prime}(N_t^-) - \left(-\mathcal{F}^{+\prime\prime}(n_{t+1}^+ + N_t^-)\right) \end{aligned} \quad (55)$$

This inequality holds in the opposite direction only when $-\mathcal{F}^{+\prime\prime}(An_t^+ + AN_{t-1}^-)$ is sufficiently larger than $-\mathcal{F}^{-\prime\prime}(AN_t^-)$, i.e. the magnitude of the slope of the marginal product of labor function is sufficiently larger in a boom than in a recession when $A > 1$.

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