

# THE EFFECTIVENESS OF EMPLOYMENT VOUCHERS: A SIMPLE APPROACH

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ABSTRACT. This paper explores the optimal design of subsidies for hiring unemployed workers (“employment vouchers” for short) in the context of a dynamic model of the labor market. Focusing on the short-term and long-term effects of the vouchers on employment and unemployment, the analysis shows how the optimal policy depends on the rates of hiring and firing, and on the problems of displacement and deadweight. It also examines the roles of the government budget constraint and of the level of unemployment benefits in optimal policy design. We calibrate the model and evaluate the effectiveness of employment vouchers in reducing unemployment for a wide range of feasible parameters.

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## 1. INTRODUCTION

Over the past two decades, subsidies for hiring unemployed workers have become an increasingly favored tool for dealing with unemployment. The subsidies may be granted to employers or employees and they may be implemented through a wide variety of policy instruments, such as tax breaks, grants, and so on. Since these policies all have analogous effects on labor market activities and government budgetary outlays, this paper groups them together under the broad heading of “employment vouchers”.

Employment vouchers have some well-known advantages in comparison with other policy instruments to tackle high unemployment. First, the vouchers are an appropriate way of dealing with a wide variety of market failures that lead to excessive real wages and thereby depress labor demand. When the cost of labor is inefficiently high, employment vouchers are a straightforward instrument to reduce labor costs, regardless of whether the excessive costs are due to, say, efficiency wage, insider-outsider, or labor union considerations. Second, employment vouchers operate as an automatic stabilizer in the labor market, in contrast to discretionary subsidies to groups of workers with particular characteristics. For example, if unskilled service sector workers have the highest unemployment rate initially and subsequently unemployment rises among semiskilled manufacturing labor, then the targeting of the employment vouchers will automatically shift from the first group to the second.

This paper explores the optimal design of employment vouchers in the context of a simple, empirically implementable, dynamic macroeconomic model of the labor market. We aim to analyze the short- and long-term effects of this policy on employment and unemployment, identify the major channels whereby this policy works, examine the main obstacles inhibiting the effectiveness of the policy, and investigate the role of the government budget constraint on policy formation.

Most theoretical and empirical studies about the effectiveness and desirability of employment vouchers have been conducted in the context of static analytical frameworks.<sup>1</sup> At best, work along these lines can isolate only the short-run effects of the policy. As our analysis below shows, however, employment vouchers can be expected to have dynamic effects, and these effects are likely to lead to outcomes that differ significantly from what may be expected to occur in the short run. The main reason is that incumbent employees' probabilities of being retained usually exceed the unemployed workers' probabilities of being hired. Consequently, provided that employment vouchers stimulate hiring more than firing, they improve people's longer-run job prospects and these long-term effects can dwarf the short-term ones.

The existing macro literature on subsidizing employment has also tended to ignore the full effects of these subsidies on the government's budget. It is standard to assume that the aggregate amount the government spends on the subsidies must be equal to its aggregate tax receipts, e.g. receipts from payroll taxes.<sup>2</sup> This approach is seriously incomplete, for a major cost of unemployment to the government comes from unemployment benefits and other associated welfare state entitlements, and when the subsidies reduce unemployment, the resulting reduction in the government's unemployment benefit payments must be included in the government's budget constraint as well.

Furthermore, the literature on empirical evaluations of employment voucher schemes tend to focus on just two factors limiting the effectiveness of this policy: deadweight (vouchers given to people who would have found jobs

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<sup>1</sup>See, for example, ((Layard, Nickell, and Jackman 1991), pp. 490-2) and (Snower 1994). It is also common for evaluations of this policy to be conducted within a static framework of analysis (e.g. Institute for Employment Studies (1994), Hamblin Research (1996), NERA (1995, 1997), Woodbury and Spiegelman (1987)). Some dynamic analyses are quoted below, but they suffer from other deficiencies, to be covered presently.

<sup>2</sup>See, for example, (Layard, Nickell, and Jackman 1991), p. 490.

anyway) and displacement<sup>3</sup> (subsidized employees displacing current employees who are not subsidized). Our analysis shows, however, that although these factors are important, they are far from constituting a comprehensive account of the main obstacles to this policy. Thus policy makers who focus predominantly on them will gain a misleading picture of the underlying problem and will be led in inappropriate policy responses. Our analysis permits a more balanced assessment of the channels whereby employment vouchers reduce unemployment.

Our analysis concentrates on six major determinants of optimal employment vouchers: (1) *deadweight* (represented by the hiring rate in the absence of vouchers), (2) *hiring responsiveness* (the effect of vouchers on the hiring rate), (3) *autonomous job loss* (depicted by the flow from employment into unemployment in the absence of the vouchers), (4) *displacement* (represented by the effect of the vouchers on the flow from employment to unemployment), (5) *unemployment benefits*, and (6) the *budgetary allocation* for the voucher policy (the government budget deficit or surplus that is to be generated through the policy). Surprisingly enough, the existing theoretical literature on the macro-economic effects of subsidizing employment has paid scant attention to the inter-related roles of these factors in employment policy formulation.

This paper covers these important neglected issues. Our aim is to construct a model that is easy to use in the practical design of employment voucher policy. In particular, our model is meant to provide a computational framework for evaluating the effectiveness of the policy, given only a small number of empirically identifiable parameters. The existing dynamic

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<sup>3</sup>We define displacement broadly to cover not only the replacement of incumbent employees by subsidized recruits *within a particular firm*, but also *inter-firm displacement* that arises when vouchers promote employment at labor-intensive firms at the expense of dismissals in capital-intensive firms.

models<sup>4</sup> for employment policy evaluation tend to be black boxes, whose predictions depend on a larger number of microeconomic parameters that are difficult to assess. The underlying problem is that general equilibrium models derived from microeconomic foundations are generally difficult to parameterize reliably for policy prediction purposes. In this paper we avoid this difficulty by adopting a simple, empirically tractable methodology. The dynamic effects arising from the difference between retention and hiring probabilities can be captured straightforwardly through a model of the labor market in which workers' transitions between employment and unemployment are governed by a Markov process. We specify the transition probabilities as simple, identifiable functions of the employment vouchers, without specifying an underlying, full-blown choice theoretic foundations. (We do, however, provide an illustrative microfoundations model in Appendix E.) Although this methodology imposes some limits on the applicability of our results (to be discussed below), it does have the advantage of simplicity and empirical tractability.

In this context, the paper focuses on a simple, useful policy problem, namely, to *find the magnitude of employment vouchers that minimize the level of unemployment, subject to a government budget constraint*. It is with reference to this policy objective that we explore the properties of the “optimal” employment voucher. We begin by concentrating on self-financing employment vouchers, i.e. ones whose cost to the government does not exceed the corresponding amount saved on unemployment benefits. We then examine how the optimal policy is affected by a change in the government budget constraint, viz, a switch from a self-financing policy to vouchers on which the government does not spend more than a fixed amount, which could be positive (implying budget deficit from the voucher policy) or negative (implying a surplus).

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<sup>4</sup>See, for instance, Hoon and Phelps (1996), Millard and Mortensen (1997), Mortensen and Phelps (1994), Phelps and Hoon (1992), and Pissarides (1994).

The paper is organized as follows. Section 2 presents our Markov model of the labor market and describes the government's budget constraint. Section 3 focuses attention on some particularly important dynamic implications of employment vouchers by considering the simple case in which the hiring probability depends linearly on the voucher and the firing probability is constant, so that there is no displacement. In this context, we derive the optimal long-run, self-financing vouchers. This analysis highlights the role of deadweight in the design of employment vouchers. Section 4 solves the policy problem when both the hiring and firing probabilities depend linearly on the voucher. This model sheds light on the joint role of displacement and deadweight costs in subsidy design. Section 5 calibrates the model and evaluates the effectiveness of the policy for a wide range of feasible parameters. Then Section 6 derives bounds for the vouchers when the hiring and firing probabilities have more general functional forms. Section 7 moves beyond self-financing employment vouchers by deriving the optimal policy when the government runs a specified policy-induced deficit or surplus. Finally, Section 8 concludes.

## 2. THE UNDERLYING MODEL

Time is discrete and workers can be in one of two states, employment or unemployment. Let  $h$  be the probability that an unemployed worker will be hired, and  $f$  be the probability that an employed worker will become unemployed (e.g., be “fired”). The labor force  $L$  is assumed constant through time.<sup>5</sup> Let  $n_t$  be employment rate in period  $t$  (the level of employment as a fraction of the labor force) and  $u_t$  be the unemployment rate (the level of

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<sup>5</sup>This simplifying assumption is one of substance. If the employment vouchers, in raising employment, also raise the labour supply (by reducing the discouraged worker effect), then the vouchers will have a smaller effect on unemployment than they would in the absence of a labour supply response.

unemployment as a fraction of the labor force) in that period. Thus:

$$(1) \quad n_t + u_t = 1$$

**2.1. The Employment and Unemployment Equations.** The change in employment

( $\Delta n_t = n_t - n_{t-1}$ ) is the difference between the number of people hired and the number of people fired:  $\Delta n_t = hu_{t-1} - fn_{t-1}$ . Obversely, the change in unemployment  $\Delta u_t = u_t - u_{t-1}$  is  $\Delta u_t = fn_{t-1} - hu_{t-1}$ . Thus the evolution of the labour market may be described by the following system:

$$(2) \quad S_t = TS_{t-1}$$

where  $S_t$  is a vector of labor market states:

$$(2.A) \quad S_t = \begin{pmatrix} n_t \\ u_t \end{pmatrix}$$

and  $T$  is the Markov matrix of transition probabilities:

$$(2.B) \quad T = \begin{pmatrix} 1-f & h \\ f & 1-h \end{pmatrix}$$

(Illustrative microfoundations for the hiring and firing rates are given in Appendix E.)

We now turn to the effect of employment vouchers on this system. We assume that each unemployed worker receives the same employment voucher, granted for one period. Let  $v$  be the ‘‘voucher ratio,’’ i.e., the ratio of the employment voucher to the wage.<sup>6</sup> An increase in the voucher ratio stimulates both hiring and firing:

$$(3) \quad h = h(v) \quad h'(v) > 0$$

$$(4) \quad f = f(v) \quad f'(v) \geq 0$$

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<sup>6</sup>In what follows, all incentives to hire and fire will be specified relative to the wage.

(Firing is encouraged through the displacement of incumbent employees by subsidized new entrants.<sup>7</sup>) These hiring and firing functions are reduced forms; they represent the degree to which the employment voucher affects the employees' incentives to work and the firms' incentives to employ, taking into account heterogeneity of jobs and workers, self-selection bias, and so on.<sup>8</sup>

By Eqs. (1), (2), (3) and (4) above, we obtain the following employment equation, showing how the government can affect the long-run unemployment rate<sup>9</sup> by varying the voucher ratio  $v$ :

$$(11.A) \quad n(v) = \frac{h(v)}{f(v) + h(v)},$$

The corresponding unemployment equation is:

$$(11.B) \quad u(v) = \frac{f(v)}{f(v) + h(v)}.$$

The voucher stimulates steady-state (long-run) employment so long as it raises the hiring rate by proportionately more than it raises the firing rate.<sup>10</sup>

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<sup>7</sup>In practice, displacement is to some degree matter of policy choice since the policy maker can reduce displacement, say, by fining employers who can be shown to have replaced incumbent employees by new recruits. The greater the degree to which anti-displacement provisions are monitored and enforced, the less the firing rate will depend on the vouchers and, since these provisions generally raise the cost of recruitment, they also reduce the responsiveness of the hiring rate to the vouchers.

<sup>8</sup>For this reason it is unnecessary for us to specify how the hiring rates and the firing rates differ across groups of workers, e.g., incumbents versus subsidized workers.

<sup>9</sup>For practical policy purposes, focusing on the long-run steady state is not as serious a limitation as it may appear at first sight, since in general it is politically and institutionally infeasible to devise detailed rules whereby employment vouchers vary through time in response to changing labor market conditions.

<sup>10</sup>To see this, differentiate Eq. (11.A),

$$\left. \frac{\partial N}{\partial v} \right|_{LR} = \frac{[h'(v)(f(v) + h(v)) - h(v)(h'(v) + f'(v))]}{(f(v) + h(v))^2}$$

and observe that when condition (6) is satisfied, the numerator is positive.

$$(6) \quad \frac{h'(v)}{h(v)} > \frac{f'(v)}{f(v)}.$$

In what follows, we plausibly take this to be the case.

Some have argued that in the long-run that any wage subsidy leads to an equal increase in the wage that employees receive, and consequently wage subsidies have no effect on long-run labor costs or long-run employment. (For example (Layard, Nickell, and Jackman 1991) (p. 108) uses a version of this argument, applied to taxes on labor.) The argument is that if the wage is the outcome of a Nash bargain and if the subsidy falls in equal proportions on the employees' take-home pay and on their fall-back position, then the subsidy can be factored out of the Nash maximand, leaving the wage paid by the firm unchanged. This argument, however, is unlikely to hold in practice, particularly for vouchers to previously unemployed workers. First, these vouchers may be expected to induce people to move from inactivity to active job search, thus raising the supply of labor, reducing wages, and raising employment. Second, the fall-back position of previously unemployed people depends on unemployment benefits, minimum wages and welfare state entitlements, and the latter need not necessarily rise in proportion to the vouchers. Third, in the transition to the long run, the vouchers may be expected to raise the number of employees relative to the number of unemployed people, and if (as is generally the case) the retention rate of employees tends to exceed the hiring rate for the unemployed (at any given real wage) the vouchers will then raise the long-run employment rate. Finally, the vouchers will generally raise the recruits' take-home pay relative to their non-wage income and thereby induce them to work harder, shirk less, and be less motivated to quit, thereby reducing the profit-maximizing efficiency wage. For these various reasons, we will assume here that there is a positive long-run equilibrium relation between the hiring rate and the magnitude of the employment voucher.

**2.2. The Government Budget Constraint.** As noted, the government's policy problem is to find the magnitude of the voucher that minimizes the unemployment rate, subject to the government budget constraint. We specify this constraint straightforwardly as follows. Let  $u(v)$  be the long-run unemployment rate as a function of the voucher ratio  $v$ . Then, since the number of unemployed people hired in each period is  $h(v)u(v)$ , the "voucher cost" (total cost of vouchers to the government) is  $v h(v)u(v)$ .

This cost must be set against the "voucher revenue", which is the total amount that the government saves on unemployment benefits due to the voucher-induced rise in the employment rate. Let  $u(v)$  and  $u(0)$  be the long-run unemployment rates in the presence and absence of the voucher, respectively. Let the replacement ratio (the ratio of the unemployment benefit to the wage)  $b$  be a positive constant. Then the amount that the employment vouchers enable the government to save on unemployment benefit disbursements is  $b(u(0) - u(v))$ .

Finally, let  $G$  be the maximum lump-sum cost per capita of the employment policy to the government, per capita, relative to the wage  $w$ . ( $G$  could be positive, zero or negative.) Consequently, the government budget constraint (GBC) is:

$$(7) \quad v h(v)u(v) \leq G + b(u(0) - u(v))$$

i.e. the per capita cost of the employment vouchers  $v h(v)u(v)$  must not exceed the maximum per capita cost of the policy to the government  $G$  plus voucher revenue  $b(u(0) - u(v))$  from reduced unemployment.

Net government spending on the employment vouchers,  $v hU + b(u(v) - u(0))$ , need not be *not* monotonic in  $v$ : At low enough levels of the voucher  $v$  (and high enough levels of the replacement ratio  $b$ ) a rise in the voucher may actually *reduce* government spending on the vouchers, since the rise in the voucher may reduce employment sufficiently and to generate more voucher revenue  $b(u(0) - u(v))$  than voucher cost ( $vhU$ ). But provided that voucher cost rises faster with the level of the voucher than does

voucher revenue, then at higher levels of the voucher ratio (and lower levels of the replacement ratio) an increase in the voucher ratio will of course raise government spending, and at the policy optimum — when unemployment is minimized — the optimal employment voucher ratio  $v = v^*$  is such that the government budget constraint Eq. (7) holds as an equality:

$$(7') \quad v^* h(v^*) u(v^*) = b(u(0) - u(v^*)) + G.$$

Observe that the greater is net government spending  $G$ , the greater is the maximum voucher ratio that satisfies the government budget constraint, and consequently the greater is the employment rate generated by the policy. In what follows, we will assume that  $G$  is sufficiently large ( $G \geq \bar{G}$ ) so that there exists a voucher ratio  $v > 0$  such that the government budget constraint is satisfied.<sup>11</sup>

Then, by the employment equation Eq. (11.A) and the government budget constraint Eq. (7'), the optimal voucher ratio ( $v^*$ ) is given by:

$$(8) \quad \left[ \frac{h(v^*)}{f(v^*) + h(v^*)} \right] = \frac{v^* h(v^*) + bn(0) - G}{b + v^* h(v^*)}.$$

Eq. (8) defines optimal voucher policy implicitly. In the next two sections, we derive the optimal policy explicitly for particular parametric forms of the hiring function  $h(v)$  and firing function  $f(v)$ .

### 3. OPTIMAL EMPLOYMENT VOUCHERS IN THE ABSENCE OF DISPLACEMENT

We focus on the case of balanced-budget voucher programs:<sup>12</sup>  $G = 0$ . We assume that

- hire rates are given by the linear equation:

$$(15.A) \quad h(v) = \eta_0 + \eta_1 v, \quad \eta_0 > 0 \quad \eta_1 > 0$$

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<sup>11</sup>We have already assumed that Eq. (6) holds. In this case, the optimal voucher reduces unemployment.

<sup>12</sup>The case of  $G > 0$  involves more algebra which can obscure some of the economic insights; thus, we will deal with it separately in Section 7.

- fire rates are independent of the voucher ratio (i.e., there is no displacement) :

$$(15.B) \quad f(v) = \lambda_0, \quad \lambda_0 > 0$$

where “ $\eta$ ” stands for “employment” and “ $\lambda$ ” stands for “job loss”. The coefficient  $\eta_0$  stands for *deadweight* (the hiring rate in the absence of the voucher),  $\eta_1$  is *hiring responsiveness* (the effect of the voucher on the hiring rate); and  $\lambda_0$  is *autonomous job loss* (the rate at which employees become unemployed).

**3.1. The Optimal Voucher.** Substituting the hiring function Eq. (15.A) and the firing function Eq. (15.B) into the unemployment equation Eq. (11.B), we obtain the relation between unemployment and the voucher ratio:

$$(10) \quad u(v) = \frac{\lambda_0}{\lambda_0 + \eta_0 + \eta_1 v}.$$

Thus, the *voucher cost* may be expressed as:

$$(11) \quad v \cdot h(v)u(v) = v \cdot (\eta_0 + \eta_1 v) \cdot \frac{\lambda_0}{\lambda_0 + \eta_0 + \eta_1 v}$$

and the *voucher revenue* becomes:

$$(12) \quad b \cdot [u(0) - u(v)] = b \cdot \left[ \frac{\lambda_0}{\lambda_0 + \eta_0} - \frac{\lambda_0}{\lambda_0 + \eta_0 + \eta_1 v} \right].$$

For a balanced budget policy ( $G = 0$ ), the government budget constraint simply provides that the voucher cost must not exceed voucher revenue (by (11) and (12)):

$$(13) \quad v \cdot (\eta_0 + \eta_1 v) \cdot \frac{\lambda_0}{\lambda_0 + \eta_0 + \eta_1 v} \leq b \cdot \left[ \frac{\lambda_0}{\lambda_0 + \eta_0} - \frac{\lambda_0}{\lambda_0 + \eta_0 + \eta_1 v} \right].$$

Since the voucher stimulates employment,<sup>13</sup> the optimal voucher (at an interior optimum<sup>14</sup>) satisfies this constraint with an equality. Expressing these terms as magnitudes in units of the voucher ratio per person unemployed (i.e., dividing both sides of Eq. (13) by  $v$  and by  $\left(\frac{\lambda_0}{\lambda_0 + \eta_0 + \eta_1 v}\right)$ ), the government budget constraint becomes:

$$(14) \quad \eta_0 + \eta_1 v = \frac{b \eta_1}{\lambda_0 + \eta_0}$$

i.e., the voucher cost (per unemployed person, in voucher units) is not greater than the voucher revenue (measured in the same terms).

The voucher cost may be divided into two components: (i) *deadweight*,  $\eta_0$  (the cost of providing vouchers for people who would have become employed anyway) and (ii) the voucher cost of induced hiring  $\eta_1 v$ . By Eq. (14), this means that the voucher cost of induced hiring must not exceed voucher revenue minus deadweight:

$$(15) \quad \eta_1 v \leq \frac{b \eta_1}{\lambda_0 + \eta_0} - \eta_0$$

Thus, the optimal voucher ratio for the long-run steady state is:<sup>15</sup>

$$(16) \quad v^* = \frac{b}{\lambda_0 + \eta_0} - \frac{\eta_0}{\eta_1}$$

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<sup>13</sup>By Eq. (1) and Eq. (10), the employment equation becomes:

$$n(v) = \frac{\eta_0 + \eta_1 v}{\lambda_0 + \eta_0 + \eta_1 v}$$

Differentiating this equation, we find that the voucher ratio stimulates employment:

$$\frac{\partial N}{\partial v} = \frac{\eta_1 \lambda_0}{(\lambda_0 + \eta_0 + \eta_1 v)^2} > 0$$

<sup>14</sup>There are two further constraints on the size of the voucher, namely a non-negativity constraint:  $v \geq 0$ , and a constraint specifying that the hiring rate cannot exceed unity ( $h = \eta_0 + \eta_1 v \leq 1$ ), so that  $v \leq \frac{1 - \eta_0}{\eta_1}$ .

<sup>15</sup>(Orszag and Snower 1997) achieve the same result for a much more complex model involving an infinite number of states but constant transition rates.

Thus, for the linear hiring function (15.A) and the constant firing rate (15.B), the optimal voucher ratio<sup>16</sup>

- rises at an increasing rate with hiring responsiveness  $\eta_1$  and
- falls at an increasing rate with deadweight  $\eta_0$  and autonomous job loss  $\lambda_0$ .

The intuition underlying these results is clarified in Figs.(1). Here the  $C$  curve represents voucher cost (Eq. (11)) and the  $R$  curve stands for voucher revenue (Eq. (12)), both as a function of the voucher ratio. The voucher ratio that minimizes unemployment is the maximal voucher for which voucher revenue does not fall short of voucher cost. Thus the optimal voucher lies at the intersection of the  $C$  and  $R$  curves.

An *increase in deadweight* shifts the voucher cost curve upwards, since this causes more people to qualify for the voucher. It also shifts the voucher revenue curve downwards, since it reduces the difference between unemployment in the absence and presence of the voucher. Consequently, as shown in Fig. (1a) the optimal voucher ratio falls.

An *increase in hiring responsiveness* raises voucher revenue (since unemployment in the presence of the voucher rises relative to unemployment in its absence) and raises the voucher cost curve (since more people get the voucher); however, the former effect dominates (see Appendix A for the details) so that the optimal voucher ratio increases as shown in Fig. (1b).

Along the same lines, an *increase in autonomous job loss* raises the voucher cost curve (since some of the extra people who lose their jobs get the voucher) and also raises the voucher revenue curve (since it increases the difference between unemployment in the absence and presence of the voucher). As shown in Appendix A, the former effect dominates so that the optimal voucher ratio decreases as illustrated in Fig. (1c).

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<sup>16</sup>In the context of our analysis, it is not possible to assess the influence of the replacement ratio  $b$  on the optimal voucher, since we have not specified how this replacement ratio affects the hiring and firing functions.

**3.2. The Short-run versus Long-run Effectiveness of the Policy.** It is worth noting that our analysis of voucher effectiveness is starkly at odds with a large body of empirical evaluations undertaken in various OECD countries ((NERA) 1997), ((NERA) 1995), (Martin Hanblin Research 1996), (Woodbury and Spiegelman 1987) and (Institute for Employment Studies 1994)). The standard practice in these evaluations is to measure the effectiveness of employment subsidies by seeking the following statistics: how many people in the targeted group got jobs within a limited period of time (typically a quarter or a year), how many of these people would have gained employment without the subsidy within that period, how many incumbent employees (outside the target group) were displaced by subsidized workers within that period, and how many non-employed people outside the target group were left jobless within that period even though they would have found jobs in the absence of the subsidy. This approach focuses on the *short-run effects* of the policy, largely ignoring the dynamic repercussions in the longer run. Although the empirical evaluations do occasionally distinguish between short-run and long-run elasticities of labor demand, they generally do not examine - as our analysis here has done - the effects of the policy on the transition rates between employment and unemployment, and thus they are unable to evaluate the effects of the policy once the associated lagged adjustment processes have worked themselves out.

It is interesting to examine the nature of this bias. Do the empirical evaluations tend to over-estimate or under-estimate the long-run effects of the policy? To shed light on this issue, we will examine two features: (i) the difference between the short-run and long-run employment effects of a given voucher ratio and (ii) the difference between the short-run and long-run self-financing, unemployment-minimizing voucher ratio. We will show how deadweight ( $\eta_0$ ), autonomous job loss ( $\lambda_0$ ), and hiring responsiveness ( $\eta_1$ ) influence these features.

The difference between the short-run and long-run employment effects of a given voucher may be derived straightforwardly as follows. By Eqs. (2), (15.A) and (15.B), the short-run employment rate is:

$$\begin{aligned} n_t &= h \cdot u_{t-1} + (1 - f) \cdot n_{t-1} \\ &= (\eta_0 + \eta_1 v) u_{t-1} + (1 - \lambda_0) n_{t-1} \end{aligned}$$

Thus, the short-run effect of the voucher (i.e. the effect in the first period) is  $\partial N_t / \partial v = \eta_1 u_{t-1}$ . To make this effect comparable with its long-run counterpart, we evaluate both at the long-run unemployment rate. Thus evaluated, the short-run effect becomes:

$$(17) \quad \left. \frac{\partial N}{\partial v} \right|_{SR} = \frac{\eta_1 \lambda_0}{\eta_0 + \eta_1 v + \lambda_0}$$

The long-run effect is:<sup>17</sup>

$$(18) \quad \left. \frac{\partial N}{\partial v} \right|_{LR} = \frac{\eta_1 \lambda_0}{(\eta_0 + \eta_1 v + \lambda_0)^2}$$

By Eq. (18) and Eq. (17), the difference between the long- and short-run effects of a given voucher is

$$(19) \quad \left. \frac{\partial N}{\partial v} \right|_{LR} - \left. \frac{\partial N}{\partial v} \right|_{SR} = \frac{\eta_1 \lambda_0}{(\lambda_0 + \eta_0 + \eta_1 v)^2} - \frac{\eta_1 \lambda_0}{\lambda_0 + \eta_0 + \eta_1 v}$$

which is always positive, so long as the retention rate ( $1 - f = 1 - \lambda_0$ ) exceeds the hiring rate ( $h = \eta_0 + \eta_1 v$ ).<sup>18</sup> In other words, the empirical evaluations above must *under-estimate* the employment effect of the policy.

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<sup>17</sup>The long-run employment rate is:

$$n = \frac{\eta_0 + \eta_1 v}{\eta_0 + \eta_1 v + \lambda_0}$$

<sup>18</sup>This result does not depend on the assumption that the hiring rate is linear and that the firing rate is a constant. For the general functional forms  $h = h(v)$ ,  $h'(v) > 0$  and  $f = f(v)$ ,  $f'(v) > 0$ , the short-run employment rate is  $n_t = h(v)u_{t-1} + (1 - f(v))n_{t-1}$  (by Eqs. 2), so that short-run effect of the voucher (starting from a steady state, using Eq.

The previous equation indicates that the difference between the long-run and short-run employment effects will be greater

- the greater is the deadweight  $\eta_0$ .
- the greater is the autonomous job loss ( $\lambda_0$ ), and
- the greater is the hiring responsiveness ( $\eta_1$ ).

Now consider the difference between the optimal voucher in the long- and short-run. Substituting the short-run unemployment rate  $u_t(v) = f(v)n_{t-1} + (1 - h(v))u_{t-1}$  into the government budget constraint Eq. (7), setting  $G = 0$ , and evaluating the expression at the steady state where  $n_{t-1} = n(0)$  and  $u_{t-1} = u(0)$ , we obtain the short-run government budget constraint:

$$(20) \quad vh(v)u(0) = bu(0) - bu_t(v).$$

Furthermore, substituting the hiring function Eq. (15.A) and the firing function Eq. (15.B) into Eq. (20), we obtain:

$$(21) \quad vh(v) - \eta_1vb = 0$$

The optimal short-run voucher is:<sup>19</sup>

(11.A) and Eq. (11.B)) is:

$$\frac{\partial N}{\partial v} \Big|_{SR} = \frac{h'(v)f(v) - f'(v)h(v)}{f(v) + h(v)}.$$

By Eq. (11.A) the long-run effect is:

$$\frac{\partial N}{\partial v} \Big|_{LR} = \frac{h'(v)f(v) - f'(v)h(v)}{(f(v) + h(v))^2}.$$

If condition (6) holds (so that the proportional increase in hiring exceeds the proportional increase in firing in response to the voucher), then the long-run and short-run employment effects will both be positive. Furthermore, if the retention rate  $(1 - f(v))$  exceeds the hiring rate  $h(v)$  — which holds whenever current employees have some degree of job security which currently unemployed people do not share — the long-run employment effect of the voucher will exceed the short-run effect.

<sup>19</sup>This is an interior optimum, assuming that the non-negativity constraint  $v \geq 0$  and the upper bound on the hiring rate ( $h \leq 1$  which implies that  $v \leq \frac{1-\eta_0}{\eta_1}$ ) are redundant.

$$(22) \quad v^0 = b - \frac{\eta_0}{\eta_1}$$

Assuming that the retention rate exceeds the hiring rate, observe that the short- and long-run vouchers have the same voucher cost but the long-run voucher yields greater voucher revenue than the short-run voucher. Thus, the long-run voucher exceeds the short-run voucher:

$$(23) \quad v^* - v^0 = b \cdot \frac{(1 - \lambda_0 - \eta_0)}{(\lambda_0 + \eta_0)} > 0.$$

This equation indicates that the voucher differential ( $v^* - v^0$ ) is greater:

- the smaller is the deadweight  $\eta_0$  and
- the smaller is the firing rate in the absence of the voucher ( $\lambda_0$ ).

#### 4. THE OPTIMAL VOUCHER IN THE PRESENCE OF DISPLACEMENT

We now consider the influence of displacement on the optimal employment voucher policy. For this purpose, we amend the firing function to make the firing rate (like the hiring rate equation 15.A) depend positively and linearly on the size of the voucher:

$$(15.B') \quad f(v) = \lambda_0 + \lambda_1 v$$

where  $\eta_0$ ,  $\eta_1$ ,  $\lambda_0$  and  $\lambda_1$  are positive constants.

The usual definition of policy-induced displacement is simply the number of people who lose their jobs on account of the policy. Our dynamic analysis offers a richer account of displacement than is possible within the standard static framework, since it draws attention to the important fact that when the hiring rate ( $h$ ) of the unemployed is less than the retention rate ( $1 - f$ ) of the employed, displacement in the short run will be greater than displacement in the long run.

In the short run, displacement may be measured by the policy-induced change in the probability that a currently employed person will be fired:

$$(24) \quad \frac{df(v)}{dv} = \lambda_1 > 0.$$

The corresponding measure is the policy-induced change in the probability that a person will be unemployed in the long run steady state:<sup>20</sup>

$$\frac{du(v)}{dv} = \frac{f'h - h'f}{(h + f)^2}$$

Under our assumption that the voucher has greater proportional effect on hiring than on firing (condition (6)), this magnitude is *negative*. In the analysis that follows, however, we will stick to the short-run definition of displacement, measuring it by the parameter  $\lambda_1$ .

**4.1. The Optimal Voucher.** The long-run unemployment rate is:

$$(25) \quad u(v) = \frac{\lambda_0 + \lambda_1 v}{\lambda_0 + \lambda_1 v + \eta_0 + \eta_1 v}$$

by Equations (11.B), (15.A) and (15.B'). Thus the voucher cost is:

$$(26) \quad vh(v)u(v) = v \cdot (\eta_0 + \eta_1 v) \cdot \frac{\lambda_0 + \lambda_1 v}{\lambda_0 + \lambda_1 v + \eta_0 + \eta_1 v}.$$

and the voucher revenue is:

$$(27) \quad b[u(0) - u(v)] = b \left[ \frac{\lambda_0}{\lambda_0 + \eta_0} - \frac{\lambda_0 + \lambda_1 v}{\lambda_0 + \lambda_1 v + \eta_0 + \eta_1 v} \right].$$

Thus, expressing both voucher revenue and voucher cost as magnitudes in units of the voucher ratio per unemployed person and assuming (as above) that the constraints  $v \geq 0$  and  $h \leq 1$  are redundant,<sup>21</sup> the government budget constraint (under a balanced budget policy,  $G = 0$ ) becomes:

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<sup>20</sup>When the transitions between labor market states are described by a Markov process, a person's long-run probability of being unemployed does not depend on initial employment status.

<sup>21</sup>The fire rate must also be less than one, which implies  $v \leq \frac{1-\lambda_0}{\lambda_1}$ . However, we have assumed that the proportional effect of a voucher on hiring is greater than that on firing, which implies:  $\frac{\eta_1}{\eta_0} > \frac{\lambda_1}{\lambda_0}$ . If the initial hire rate is greater than the initial fire rate (as in every major industrialized country), this condition implies that the hire rate restriction is binding first.

$$v(\eta_0 + \eta_1 v) = b \left[ \frac{\lambda_0 \eta_1 - \eta_0 \lambda_1}{(\lambda_0 + \eta_0)(\lambda_0 + \lambda_1 v)} \right]$$

by Eqs. (25) and (7').

Thus, the optimal voucher is the largest root of the equation:

$$(28) \quad \eta_1 \lambda_1 v^2 + (\eta_1 \lambda_0 + \lambda_1 \eta_0)v + \left[ \eta_0 \lambda_0 - b \frac{\lambda_0 \eta_1 - \eta_0 \lambda_1}{\lambda_0 + \eta_0} \right] = 0.$$

For sufficiently large  $b$ , the optimal voucher will be positive if:<sup>22</sup>

$$(29) \quad \frac{\eta_1}{\eta_0} > \frac{\lambda_1}{\lambda_0}$$

i.e., if the ratio of hiring responsiveness to deadweight exceeds the ratio of displacement to autonomous job loss. This relation is equivalent to (12).

Solving Eq. (28), we obtain the optimal voucher ratio explicitly:

$$(30) \quad v^* = \frac{1}{2\eta_1 \lambda_1} \left[ -(\eta_1 \lambda_0 + \lambda_1 \eta_0) + \sqrt{Z} \right]$$

where

$$(30.A) \quad Z = (\eta_1 \lambda_0 + \lambda_1 \eta_0)^2 - 4\eta_1 \lambda_1 \left[ \eta_0 \lambda_0 - b \frac{\lambda_0 \eta_1 - \eta_0 \lambda_1}{\lambda_0 + \eta_0} \right].$$

As shown in Appendix B and Section 5, the optimal voucher ratio

- rises at a decreasing rate (rather than increasing) rate with hiring responsiveness  $\eta_1$  (for the central estimates of the other parameters),
- falls at a decreasing (rather than increasing) rate with deadweight  $\eta_0$  and autonomous job loss  $\lambda_0$  (for the central estimates of the other parameters), and
- first rises and then falls with displacement (for the central estimates of the other parameters).

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<sup>22</sup>Eq. (6) implies:

$$\frac{\eta_0 + \eta_1 v}{\eta_1} < \frac{\lambda_0 + \lambda_1 v}{\lambda_1}$$

which is equivalent to Eq. (29).

**4.2. The Short-Run versus Long-Run Effectiveness of the Policy.** It can be shown that in the presence of displacement (as in its absence), the long-run employment effects of balanced budget vouchers are typically larger than short-run effects. In fact, as we will see below, for a wide range of feasible parameter values, the optimal, self-financing voucher ratios are zero in the short run, but significantly positive in the long run.

It is straightforward to show that the unemployment effect of a given voucher ratio is greater in the long run than in the short run. By Eqs. (2), (15.A) and (15.B'), the short-run employment rate with displacement is:

$$\begin{aligned} n_t &= h \cdot u_{t-1} + (1 - f) \cdot n_{t-1} \\ &= (\eta_0 + \eta_1 v) u_{t-1} + (1 - \lambda_0 - \lambda_1 v) n_{t-1} \end{aligned}$$

Thus, the employment effect of the voucher in the first period is  $\partial N_t / \partial v = \eta_1 u_{t-1} - \lambda_1 n_{t-1}$  and evaluating this expression at the long-run unemployment rate, we obtain:

$$(31) \quad \left. \frac{\partial n}{\partial v} \right|_{SR} = \frac{\eta_1 \lambda_0 - \lambda_1 \eta_0}{\eta_0 + \eta_1 v + \lambda_0 + \lambda_1 v}$$

The long-run employment effect under displacement is:

$$\left. \frac{dn}{dv} \right|_{LR} = \frac{\eta_1 \lambda_0 - \lambda_1 \eta_0}{(\eta_0 + \eta_1 v + \lambda_0 + \lambda_1 v)^2}$$

.

Thus, we find (once again) that the long-run effect exceeds the short-run effect if the denominator is less than one (which occurs under our assumption that the retention rate exceeds the hire rate).

Next, it can be shown that the optimal voucher ratio is greater in the long run than in the short run. In the short-run, applying the government budget constraint Eq. (20) for a balanced budget voucher we obtain:

$$(32) \quad v(\eta_0 + \eta_1 v) \frac{\lambda_0}{\eta_0 + \lambda_0} \leq b[\eta_0 + \eta_1 v] \frac{\lambda_0}{\eta_0 + \lambda_0} - b(\lambda_0 + \lambda_1 v) \frac{\eta_0}{\eta_0 + \lambda_0}$$

Simplifying, we obtain, for  $v \neq 0$ :

$$(33) \quad \eta_0 + \eta_1 v = b\eta_1 - b\lambda_1 \frac{\eta_0}{\lambda_0},$$

which implies the following optimal short-run voucher:

$$(34) \quad v^0 = b - \frac{\eta_0}{\eta_1} - b \frac{\lambda_1}{\lambda_0} \frac{\eta_0}{\eta_1}$$

Since displacement acts primarily to reduce the revenue from the replacement ratio  $b$ , the short-run voucher (34) is smaller than the short-run voucher (22) without displacement ( $\lambda_1 = 0$ ). Moreover, in the presence of displacement, the optimal long-run voucher ratio exceeds the optimal short-run voucher ratio ( $v^* > v^0$ ).

Appendix C sheds some light on the properties of optimal employment vouchers in more general contexts where the hire and fire rates are nonlinear functions of the voucher. Appendix D extends the analysis above by considering voucher policies that generate a specified net deficit or surplus to the government (rather than being self-financing).

## 5. EVALUATING THE EFFECTIVENESS OF THE POLICY

We now evaluate the effectiveness of the policy by calibrating our model and deriving the influence of the optimal voucher on unemployment for a wide range of feasible parameters. Our analysis shows that, for parameter ranges centered on values that appear reasonable for EU countries, the long-run unemployment effects of the policy are substantial and these effects significantly exceed the corresponding short-run effects.

Let the period of analysis be one quarter. A reasonable estimate for average job tenure in the EU is about 10 years (40 quarters) (c.f., (Burgess and Rees December 1994), (Simon Burgess and Rees October 1997)). With constant separation rates, a job duration estimate of 60 quarters translates

into an autonomous job loss ( $\lambda_0$ , the separation rate in the absence of vouchers) of 1/60 or 0.025.

For unemployment, the OECD reports fractions of workers who have been unemployed for over a year (4 quarters). As shown in Appendix G, if the transition rate out of unemployment is  $h$ , then the steady state proportion of people who are unemployed for more than  $x$  periods is  $(1 - h)^x$ . Thus, the fraction of the unemployed who are unemployed for more than a year is  $(1 - h)^4$ . The OECD figures suggest that a central estimate is that 33% of the unemployed have been jobless for over a year:  $(1 - h)^4 = 0.33$ .<sup>23</sup>

The central estimate of the replacement ratio in our calibration exercise is set at  $b = 0.5$  which is broadly representative of the EU. However, since the optimal voucher and the corresponding effect on unemployment rises with the replacement ratio, we use this value to derive conservative estimates of the policy's effectiveness.

We calibrate the hiring responsiveness parameter  $\eta_1$  conservatively, setting this parameter to obtain a relatively low estimate of the effects of vouchers on unemployment. Estimates of hiring elasticities in the literature (e.g. (Holzer, Katz, and Krueger 1991) and (Krueger 1988)) led Card and Krueger ((Card and Krueger 1995)) to conclude the elasticity of hiring with respect to the wage are within the range of 0.5 and 4.0.

The above estimates of the hiring elasticity are relevant, but not immediately applicable, to our model, since the elasticity above is defined with respect to permanent wage changes whereas the employment vouchers are short-lived. Thus the elasticity with respect to vouchers may be expected to be substantially than that with respect to wages. (Snower 1996) provides arguments that the voucher elasticity of employment is about one third of the corresponding wage elasticity of employment. The ratio of these two elasticities will typically be smaller than the ratio of the voucher elasticity

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<sup>23</sup>The OECD Employment Outlook registers a OECD average of 32.9% unemployment more than 12 months in 1998 as opposed to 34.7% in 1997.

of hiring relative to the wage elasticity of hiring, since employment depends on both hiring and separations, and a wage increase will generally reduce the separation rate while a voucher increase will increase it (on account of displacement). Thus, for our baseline calibration, we choose the conservative estimate of 0.5 for the elasticity of hiring with respect to vouchers. (Our analysis of policy effectiveness will however be conducted for a wide range of elasticities around this estimate).

Furthermore, in our baseline calibrations we set the elasticity of separations with respect to the voucher of 0.1 and then examine voucher effectiveness for a wide range of elasticities around this value. In practice, as noted, the elasticity depends on the existence of anti-displacement provisions. Direct estimates of displacement due to employment subsidies have a wide range of error.

Given the above estimates of the elasticity of hiring with respect to vouchers ( $\gamma_h$ ) and the elasticity of separations with respect to vouchers ( $\gamma_s$ ) and given our estimates of deadweight ( $\eta_0$ ) and autonomous job loss ( $\lambda_0$ ), we compute the voucher effectiveness coefficient  $\eta_1$  and displacement  $\lambda_0$  by solving the following system of equations:

$$(35) \quad \eta_1 \frac{v}{\eta_0 + \eta_1 v} = \gamma_h$$

$$(36) \quad \lambda_1 \frac{v}{\lambda_0 + \lambda_1 v} = \gamma_s$$

where these equations are definitions of the hiring and separation elasticities and the voucher is set at its optimal level.

Our baseline calibration parameters are summarized in Table 1. We focus on balanced budget policies:  $G = 0$ . In this baseline case, the optimal voucher in the steady state is 0.749 and, as a result, the steady state unemployment rate drops from 9.4% to 5.4%. However, the short-run voucher is zero, which means that self-financing policy is ineffective in the short-run.

Parameter	Meaning	Value
$\eta_0$	Deadweight	0.2421
$\lambda_0$	Separation Rate	0.025
$\eta_1$	Voucher Effectiveness	0.32332
$\lambda_1$	Displacement	0.0037097
$b$	Replacement Ratio	0.50

TABLE 1. Calibrated model parameters

In figures (2)-(6), we present the optimal voucher in the short run and long run and the associated unemployment rate, for a wide range of feasible parameter values.<sup>24</sup> The results indicate that for realistic elasticity values, a balanced-budget voucher can have a significant effect in reducing unemployment.<sup>25</sup>

Figs. (2) shows how optimal vouchers and unemployment vary with the deadweight in the short and long run. As the figures show, the policy is unable to have any effect on unemployment in the short run. In the long-run, however, voucher policy can reduce the unemployment rate by an amount ranging from 64% (a 7% percentage point drop) for low deadweight parameter of 0.17, to 55% (a 7% percentage point drop) for a deadweight parameter of 0.28.

Figs. (3) show how the unemployment falls and the optimal voucher rises with a rise in the effectiveness of employment vouchers. Once again, the policy is ineffective in the short run, but in the long-run the voucher-induced fall in the unemployment rate ranges from 13.7% (a 1.3% percentage point

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<sup>24</sup>Observe that the baseline unemployment rate (Eq. (11.B) with  $v = 0$ ) is independent of  $\eta_1, \lambda_1$ .

<sup>25</sup>We invite the reader to experiment with parameter values either using the formulae in the paper or our on-line voucher calculator. We have a Java applet available at <http://www.econ.bbk.ac.uk/vouchers/Vouchers.html> to compute optimal vouchers using the formulae in this paper.

drop) for a parameter of 0.2 to 60.7% (a 5.7% percentage point drop) for a parameter of 0.5.

Figs. (4) show how the optimal voucher and the corresponding unemployment rate rise with the autonomous job loss parameter in the presence and absence of the employment vouchers. While the policy is not self-financing in the short run, it once again appears to be effective in the long run, when the unemployment rate falls by an amount ranging from 5% (a 0.1% percentage drop) for an autonomous job loss parameter of 0.005 to 44% (a 5.5% percentage point drop) for a parameter of 0.035.

Along the same lines, Figures (5) and (6) show how the optimal voucher and corresponding unemployment rate vary with the replacement ratio and the displacement parameters. In both cases the policy has a powerful effect on the long-run unemployment rate over the entire range of parameter values. Also observe that, given the central estimates of the other parameters, the policy is not self-financing in the short-run.

## 6. CONCLUSION

Unemployment benefit systems have become a costly obligation for many governments. Since these systems can provide a substantial safety net without substantial government expenditures only when the unemployment rate is low, it is not surprising that these systems have come under attack in the two decades of high unemployment experienced in many European countries and elsewhere. What has made unemployment benefit systems particularly difficult to defend when unemployment is high is that they discourage job search and thereby augment the problem they are meant to address. The analysis of this paper has suggested an alternative approach to these systems: instead of seeing unemployment benefit payments merely as support given to people on the condition that they remain jobless, they can be used as a source of funding for employment-creating policies. We

have explored how employment vouchers, in reducing unemployment, create “voucher revenue” for the government (saving in terms of unemployment benefits) and how this revenue can be used to finance the vouchers themselves (wholly or partially). Thereby unemployment benefits become less of a drag on government finances and on labor market performance, and turn into a useful resource instead.

In recent years, policy makers have come increasingly to recognize the potential importance of subsidizing the jobs of currently unemployed people. But despite the growing interest in the design of such policies, there has been little dynamic analysis of the optimal policies and their short- and long-term employment effects. This paper provides a simple analytical framework for doing so.

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APPENDIX A: COMPARATIVE STATICS IN THE ABSENCE OF  
DISPLACEMENT

This appendix reviews some comparative statics results for the basic model without displacement which are shown graphically in Figs. (1)-(4).

### Deadweight

The effect on the voucher cost is:

$$\frac{d \ln C}{d\eta_0} = \frac{1}{\eta_0 + \eta_1 v} - \frac{1}{\lambda_0 + \eta_0 + \eta_1 v} > 0$$

by Eq. (11).

The effect on the voucher revenue (using Eq. (12) is:

$$\frac{dR}{d\eta_0} = b \left[ -\frac{\lambda_0}{(\lambda_0 + \eta_0)^2} + \frac{\lambda_0}{(\lambda_0 + \eta_0 + \eta_1 v)^2} \right] < 0$$

Thus, by Eq. (16), the overall effect is:

$$\frac{dv^*}{d\eta_0} = \frac{-b}{(\lambda_0 + \eta_0)^2} - \frac{1}{\eta_1} < 0.$$

### Hiring Responsiveness

The effect on costs (using Eq. (11) is:

$$\frac{d \ln C}{d\eta_1} = \frac{v}{\eta_0 + \eta_1 v} - \frac{v}{\lambda_0 + \eta_0 + \eta_1 v} > 0.$$

The effect on the voucher revenue is:

$$\frac{dR}{d\eta_1} = b \cdot \left[ \frac{\lambda_0 v}{(\lambda_0 + \eta_0 + \eta_1 v)^2} \right] > 0.$$

Thus, the overall effect is:

$$\frac{dv^*}{d\eta_1} = \frac{\eta_0}{\eta_1^2} > 0.$$

### Autonomous Job Loss

The effect on the voucher cost is:

$$\frac{d \ln C}{d\lambda_0} = \frac{1}{\lambda_0} - \frac{1}{\lambda_0 + \eta_0 + \eta_1 v} > 0$$

The effect on revenues (using Eq. (12) is somewhat complex. The reason for this is that a change in autonomous firing effects both the unemployment

rate with and without vouchers. The revenue expression can be rewritten in terms of employment rates:

$$(A.1) \quad R = b \cdot [E(v) - E(0)] = b \cdot \left[ -\frac{\eta_0}{\lambda_0 + \eta_0} + \frac{\eta_0 + \eta_1 v}{\lambda_0 + \eta_0 + \eta_1 v} \right].$$

Differentiating the right-hand side of Eq. (A.1):

$$(A.2) \quad \frac{dR}{d\lambda_0} = b \left[ \frac{\eta_0}{(\lambda_0 + \eta_0)^2} - \frac{\eta_0 + \eta_1 v}{(\lambda_0 + \eta_0 + \eta_1 v)^2} \right],$$

which has the sign of the term in brackets. Consider the function:

$$(A.3) \quad G(z) = \frac{z}{(a + z)^2}$$

which has derivative:

$$(A.4) \quad G'(z) = \frac{1}{(a + z)^2} \left[ 1 - \frac{2z}{a + z} \right]$$

which is positive as long as  $\frac{z}{a+z} < \frac{1}{2}$ . In our case  $z = \eta_0 + \eta_1 v$  and  $a = \lambda_0$

so the condition means the employment rate with vouchers is less than 50%.

We expect the opposite to occur and therefore the second term in brackets will be smaller and the voucher revenue curve will shift up.

Thus, the overall effect is:

$$\frac{dv}{d\lambda_0} = \frac{-b}{(\lambda_0 + \eta_0)^2} < 0$$

## APPENDIX B: COMPARATIVE STATICS IN THE PRESENCE OF DISPLACEMENT

Totally differentiating Eq. (28) with respect to  $v$  and  $\eta_0$  yields:

$$(B.1) \quad [2\eta_1 \lambda_1 v + \eta_1 \lambda_0 + \lambda_1 \eta_0] dv + \left[ \lambda_1 v + \lambda_0 + \frac{b\lambda_0(\lambda_1 + \eta_1)}{(\lambda_0 + \eta_0)^2} \right] d\eta_0 = 0$$

Since both terms in brackets in Eq. (B.1) are positive, the optimal voucher varies inversely with deadweight:  $\frac{dv}{d\eta_0} < 0$ .

Similarly:

$$(B.2) \quad [2\eta_1 \lambda_1 v + \eta_1 \lambda_0 + \lambda_1 \eta_0] dv + \left[ \eta_1 v^2 + \eta_0 v + \frac{b\eta_0}{\lambda_0 + \eta_0} \right] d\lambda_1 = 0$$

Since both terms in brackets are positive, the optimal voucher is inversely related to displacement:  $\frac{dv}{d\lambda_1} < 0$ .

## APPENDIX C: NONPARAMETRIC BOUNDS

To shed some light on the properties of optimal employment vouchers in more general contexts where the hire and fire rates are nonlinear functionals of the voucher, this section derives non-parametric bounds for the voucher, applicable for broad classes of the hiring and firing functions.

**6.1. An Upper Bound When the Maximum Hiring Rate is Known.** We denote the hire function by  $h(v)$  and the fire function by  $f(v)$ . We assume balanced budget policies. The government budget constraint is then:

$$(vh(v) + b) \frac{f(v)}{h(v) + f(v)} \leq b \frac{f(0)}{h(0) + f(0)}$$

or:

$$(vh(v) + b) f(v) \leq b \frac{f(0)}{h(0) + f(0)} (h(v) + f(v))$$

Hence:

$$\left( vf(v) - b \frac{f(0)}{h(0) + f(0)} \right) h(v) \leq bf(v) \left( \frac{f(0)}{h(0) + f(0)} - 1 \right)$$

or:

$$\left( vf(v) - b \frac{f(0)}{h(0) + f(0)} \right) h(v) \leq bf(v) \frac{-h(0)}{h(0) + f(0)}$$

The right hand side is negative and  $h(v) > 0$  which implies that the term:

$$\left( vf(v) - b \frac{f(0)}{h(0) + f(0)} \right)$$

is negative. Let  $h^* = \sup_v h(v)$  then:

$$\left( vf(v) - b \frac{f(0)}{h(0) + f(0)} \right) \leq b \frac{f(v)}{h^*} \frac{-h(0)}{h(0) + f(0)}$$

Let  $f^* = \inf_v f(v) = f(0)$  then:

$$\left( vf(v) - b \frac{f(0)}{h(0) + f(0)} \right) \leq b \frac{f(0)}{h^*} \frac{-h(0)}{h(0) + f(0)}$$

so that:

$$\begin{aligned}
 vf(v) &\leq b \frac{f(0)}{h(0) + f(0)} + b \frac{h(0)}{h^*} \frac{-f(0)}{h(0) + f(0)} \\
 (37) \qquad &\leq b \left(1 - \frac{h(0)}{h^*}\right) \frac{f(0)}{h(0) + f(0)}
 \end{aligned}$$

Since  $vf(0) < vf(v)$ :

$$v \leq b \left(1 - \frac{h(0)}{h^*}\right) \frac{1}{h(0) + f(0)}.$$

For cases in which voucher effectiveness is known, a better bound is provided below. We proceed to extend the bounds for cases in which fire rates depend on vouchers.

**6.2. An Upper Bound by the Mean Value Theorem.** By the mean value theorem for some  $\tilde{v} \in [0, v]$ :

$$f(v) = f(0) + f'(\tilde{v})v$$

Hence in Eq. (37):

$$(f(0) + f'(\tilde{v})v)v \leq b \left(1 - \frac{h(0)}{h^*}\right) \frac{f(0)}{h(0) + f(0)}$$

This implies that:

$$v \leq \frac{1}{2\tilde{f}} \left[ -f(0) + \sqrt{f(0)^2 + 4\tilde{f}b \left(1 - \frac{h(0)}{h^*}\right) \frac{f(0)}{h(0) + f(0)}} \right]$$

where  $\tilde{f} = f'(\tilde{v}) > 0$ . This reproduces our result in Section 4 that optimal vouchers grow at most with the square root of the replacement ratio.

**6.3. An Upper Bound in the Absence of Displacement.** We assume that  $f(v) = f(0)$ . By the mean value theorem again:

$$h(v) = h(0) + h'(\tilde{v})v$$

We assume that  $f(v) = f(0)$  so that fire rates are constant and do not depend on vouchers. Recalling Eq. (6.1):

$$h(v) \left[ v - \frac{b}{f(0) + h(0)} \right] \leq b \left[ \frac{-h(0)}{h(0) + f(0)} \right]$$

we have:

$$[h(0) + h'(\tilde{v})v]v - \tilde{h}v \frac{b}{f(0) + h(0)} \leq 0$$

where  $\tilde{h} = h'(\tilde{v}) > 0$ . This leads to the optimal  $v^*$ :

$$v^* \leq \max \left( 0, -\frac{h(0)}{\tilde{h}} + \frac{b}{f(0) + h(0)} \right)$$

which is the result obtained in Section 3 where  $\tilde{h}$  is the constant voucher effectiveness  $\eta_1$ .

## 7. APPENDIX D: POLICIES THAT GENERATE A DEFICIT OR SURPLUS

To keep the analysis simple, we focus on the case where vouchers do not affect firing (e.g.,  $\lambda_1 = 0$ ). In this case the government budget constraint is:

$$[v(\eta_0 + \eta_1 v) + b] \leq [\eta_0 + \eta_1 v + \lambda_0] \left[ \frac{b}{\eta_0 + \lambda_0} + \frac{G}{\lambda_0} \right].$$

This implies that the optimal voucher  $v$  is the largest root of the equation:

$$\eta_1 v^2 + \left( \eta_0 - b \frac{\eta_1}{\eta_0 + \lambda_0} - \frac{G\eta_1}{\lambda_0} \right) v - \frac{\eta_0 + \lambda_0}{\lambda_0} G$$

which is:

$$(E.1) \quad v^* = \frac{1}{2} \left[ B + \sqrt{B^2 + 4C} \right]$$

where:

$$(E.1.A) \quad B = -\frac{\eta_0}{\eta_1} + \frac{b}{\eta_0 + \lambda_0} + \frac{G}{\lambda_0}$$

$$(E.1.B) \quad C = \frac{\eta_0 + \lambda_0}{\eta_1 \lambda_0} G > 0$$

To interpret this result simply, we note that since  $C > 0$ , the optimal voucher is bounded below by  $B$ :

$$(E.2) \quad v^* \geq \frac{b}{\eta_0 + \lambda_0} - \frac{\eta_0}{\eta_1} + \frac{G}{\lambda_0}$$

which agrees with the formula in Eq. (16) when  $G = 0$ . Since the expected time workers spend employed after receiving a voucher is  $\frac{1}{\lambda_0}$ , the third term on the right hand side of Eq. (E.2) may be interpreted as a government spending multiplier.

## 8. APPENDIX E: ILLUSTRATIVE MICROFOUNDATIONS FOR THE HIRING AND FIRING RATES

The probabilities of hiring ( $h$ ) and firing ( $f$ ) have been derived from various microeconomic foundations in the literature.<sup>26</sup> Since our analysis is concerned with these probabilities only through their response to employment vouchers, we will consider only a very rudimentary derivation of these probabilities.

Suppose that the productivity  $a$  of a recently hired worker is a random variable uniformly distributed over the interval  $[\alpha_1, \alpha_2]$ , ( $\alpha_1, \alpha_2 > 0$ ) and is independently distributed across the newly hired workers. The productivity of an incumbent employee is a constant  $A$ , where  $A > \alpha$ . We let  $w$  be the real wage,  $v$  be the “voucher ratio” (the ratio of the voucher to the wage for newly hired employees), and  $c_h$  and  $c_f$  (positive constants) be the hiring and firing costs per worker as a ratio of the wage, respectively. A worker is hired whenever<sup>27</sup>  $a - w + vw - c_h w > 0$  (i.e., the entrant generates positive profit). Thus, the hiring probability (i.e., the probability that  $a > (1 - v + c_h)w$ ) is

$$(38) \quad h = \left( \frac{1}{\alpha_2 - \alpha_1} \right) [\alpha_2 - (1 - v + c_h)w].$$

An incumbent employee is fired when  $A - w < \max[(a - (1 - v)w - (c_h + c_f)w), -c_f w]$ , i.e., firing occurs when (i) the profit generated by the incumbent is less than the profit generated by the entrant, minus the labor turnover costs  $c_h + c_f$  (in which case the incumbent is replaced by an entrant)<sup>28</sup> or (ii) the loss generated by the incumbent is less than the firing cost (in which case the incumbent’s position is kept vacant). For simplicity, we assume that  $\alpha_1 > w + c_h$  so that the firing condition reduces to  $A - w < a - (1 - v - c_h - c_f)w$  and the incumbent is replaced. Thus, the firing probability (i.e., the probability

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<sup>26</sup>For example, (Phelps 1994).

<sup>27</sup>For simplicity, we assume that the firm has a one-period time horizon.

<sup>28</sup>We assume that the incumbent is fired even though he may generate positive profit, because the firm has a fixed amount of capital and a fixed capital-labor ratio.

that  $a > A - (v + c_f + c_h) \cdot w$  is

$$(39) \quad f = \frac{1}{\alpha_2 - \alpha_1} [\alpha_2 - A + (v - c_f - c_h)w].$$

Assume that wages are set before the employment decisions are made. The wages are the outcome of a Nash bargain between each firm and its incumbent employees. The firm's profit per incumbent employee under the bargaining agreement is  $(A - w)$ ; under disagreement, the firm pays  $c_f w$ ;<sup>29</sup> thus, the profit surplus is  $(A - w \cdot (1 - c_f))$ . Assuming for simplicity that the wage is set after the firm's firing decision is made,<sup>30</sup> the incumbent employee's payoff is  $w$ . Then the Nash bargaining problem is:

$$(40) \quad \max_w [w]^\mu [A - w(1 - c_f)w]^{1-\mu}$$

where  $\mu$  (a constant) is the bargaining power of the employee relative to that of the employer, and the firing rate is given by Eq. (39). Solving, we obtain the equilibrium wage:

$$(41) \quad w^* = \frac{\mu A}{1 - c_f}$$

Substituting the equilibrium wage into the hiring rate (Eq. (38), we find:

$$(42) \quad h^* = \frac{1}{\alpha_2 - \alpha_1} \left[ \alpha_2 - (1 - v + c_h) \frac{\mu A}{1 - c_f} \right]$$

Similarly, substituting  $w^*$  into the firing rate (Eq. (4)), we obtain:

$$(43) \quad f^* = \frac{1}{\alpha_2 - \alpha_1} \left[ \alpha_2 - A + (v - c_f - c_h) \frac{\mu A}{1 - c_f} \right].$$

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<sup>29</sup>For simplicity, we assume that, under disagreement, the worker produces no output, is paid no wage and engages in industrial action whose cost to the firm is high enough to make the firm indifferent between retaining and firing the worker, as in (Lindbeck and Snower 1990), for example.

<sup>30</sup>This assumption is made only for the sake of algebraic simplicity; it is not necessary for the derivation of the hiring and firing functions (44) and (45). If the wage were set before the firing decision is made, the employee's payoff would be  $(1 - f)w$  and the Nash maximand would be a third-order polynomial in  $w$ .

We note from Eq. (42) and (43) that the voucher ratio  $v$  stimulates hiring and firing. (Firing is encouraged through the displacement of incumbent employees by subsidized new entrants.<sup>31</sup>) In general we may express these effects as:<sup>32</sup>

$$(44) \quad h = h(v) \quad h'(v) > 0$$

$$(45) \quad f = f(v) \quad f'(v) \geq 0$$

Eqs. (44) and (45) could be derived for a wide variety of existing labor market models, and Eq. (38) and Eq.(39) are simply illustrative.

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<sup>31</sup>In practice, displacement is to some degree matter of policy choice since the policy maker can reduce displacement, say, by fining employers who can be shown to have replaced incumbent employees by new recruits. The greater the degree to which anti-displacement provisions are monitored and enforced, the less the firing rate will depend on the vouchers and, since these provisions generally raise the cost of recruitment, they also reduce the responsiveness of the hiring rate to the vouchers.

<sup>32</sup>In practice, the functional form will depend, in addition to the factors enumerated above, on the degree of heterogeneity in the productivities of workers and jobs.

## APPENDIX G: THE FRACTION OF LONG TERM UNEMPLOYED

This appendix derives the fraction of long term unemployed used in the calibration exercise. The difference equations for unemployment:

$$v_{t,x} = (1 - h)v_{t-1,x-1}$$

have the solution  $v_{t,x} = (1 - h)^x v_{t-x,0}$  for  $t > x$  where the term  $v_{t-x,0}$  is the number of entrants to unemployment  $x$  periods ago. The total number of unemployed  $v_x^s$  of duration  $x$  in steady state is:

$$(C.1) \quad \sum_{x=0}^{\infty} (1 - h)^x v_0^s = \frac{v_0^s}{h}$$

where  $v_0^s$  is the number of entrants to unemployment (and the superscript  $s$  denotes the steady state).

The number unemployed for duration greater than or equal to  $y$  is:

$$(C.2) \quad \begin{aligned} \sum_{x=y}^{\infty} (1 - h)^x v_0^s &= v_0^s \sum_{x=0}^{\infty} (1 - h)^{x+y} \\ &= v_0^s (1 - h)^y \sum_{x=0}^{\infty} (1 - h)^x \\ &= \frac{v_0^s (1 - h)^y}{h} \end{aligned}$$

The ratio of Eq. (C.2) to Eq. (C.1) is  $(1 - h)^y$ . This calculation assumes duration-independent transition rates and a steady state.

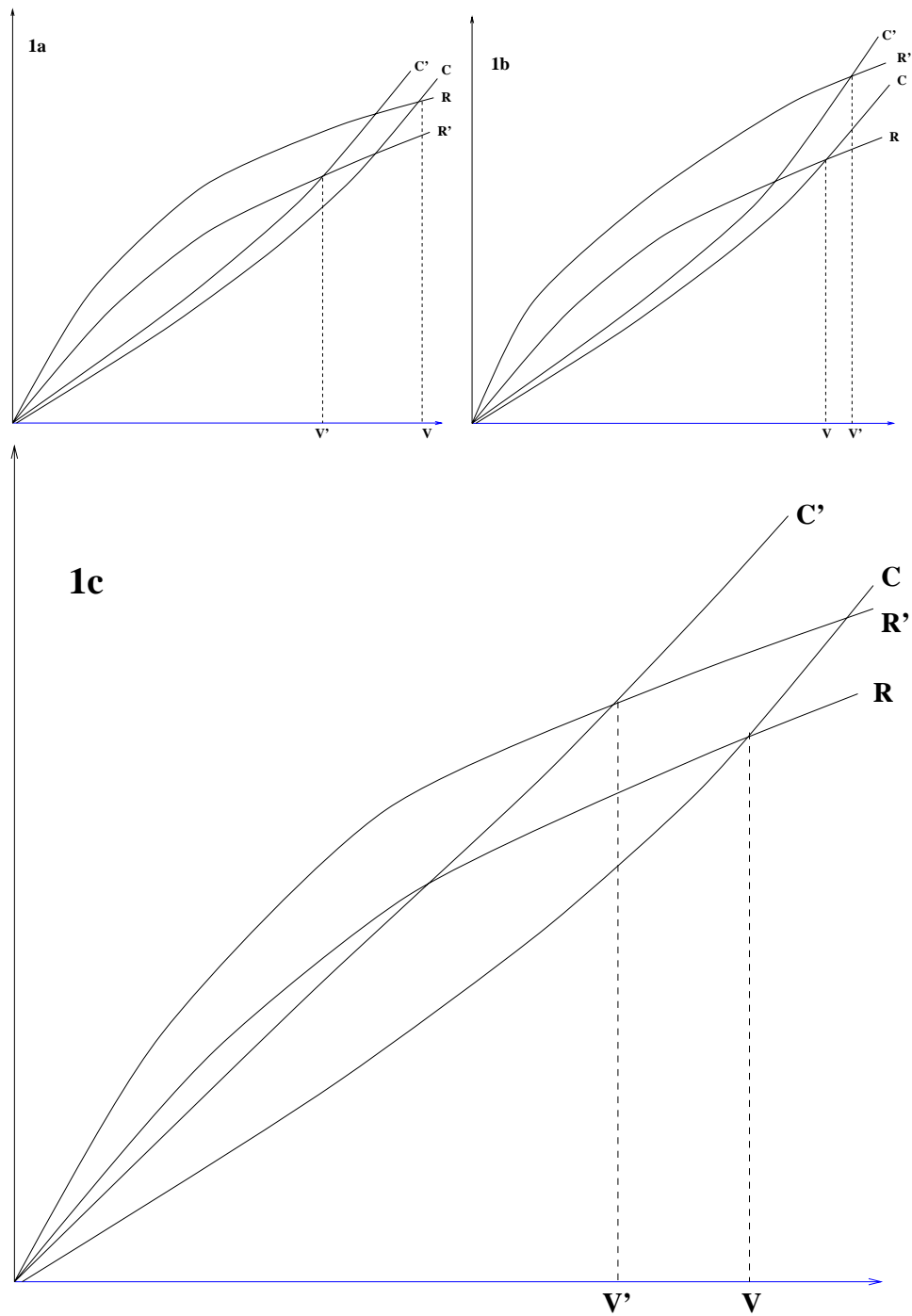


FIGURE 1. **Comparative statics.** Fig. (1a) (upper left): the effect of an increase in deadweight loss. Fig (1b) (upper right): the the effect of an increase in hiring responsiveness. Fig. (1c) (lower left): the effect of an increase in autonomous job loss.

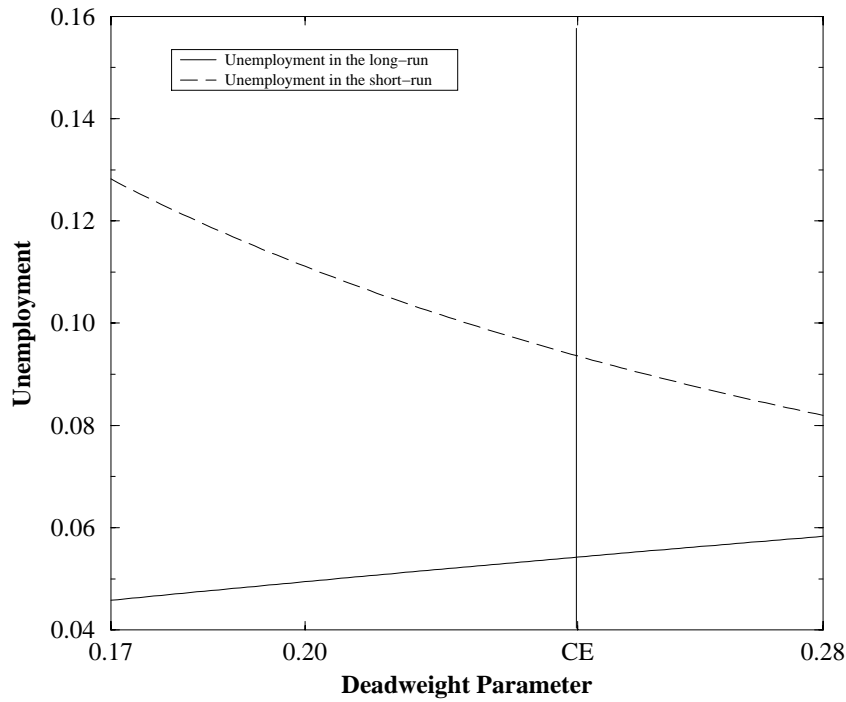
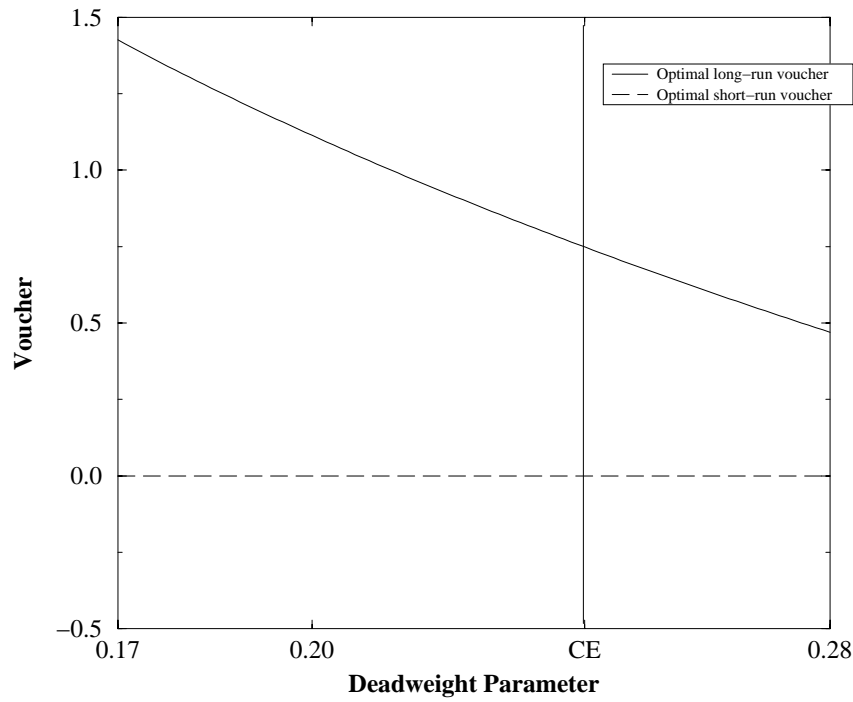


FIGURE 2. The Effects of Deadweight

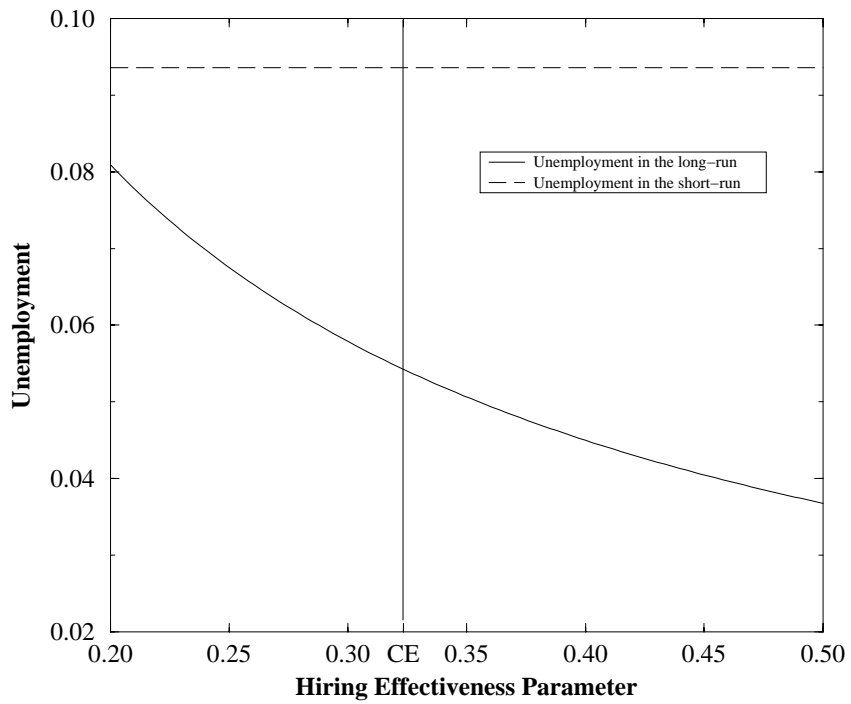
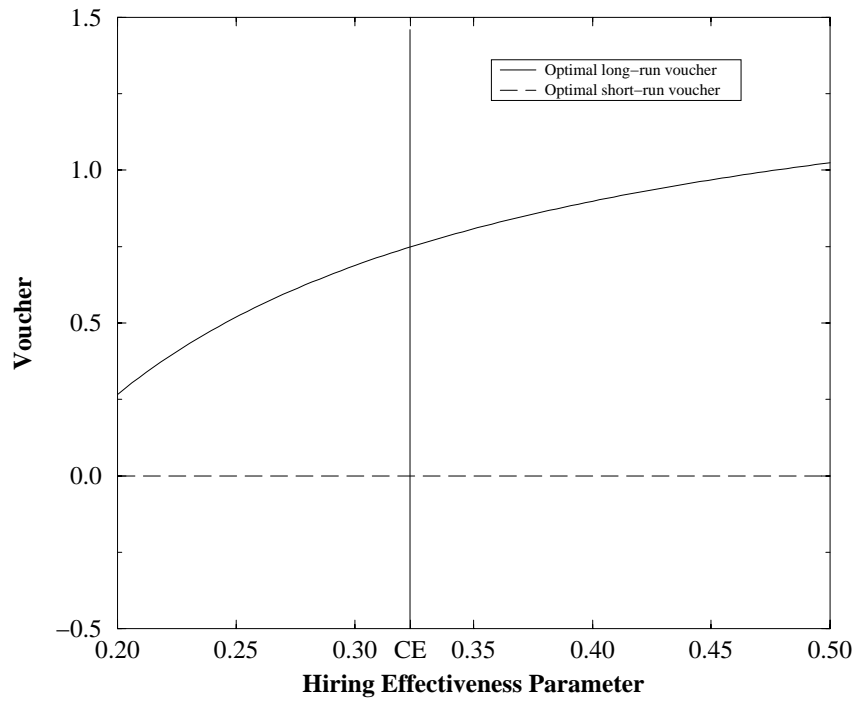


FIGURE 3. The Effects of Hiring Responsiveness

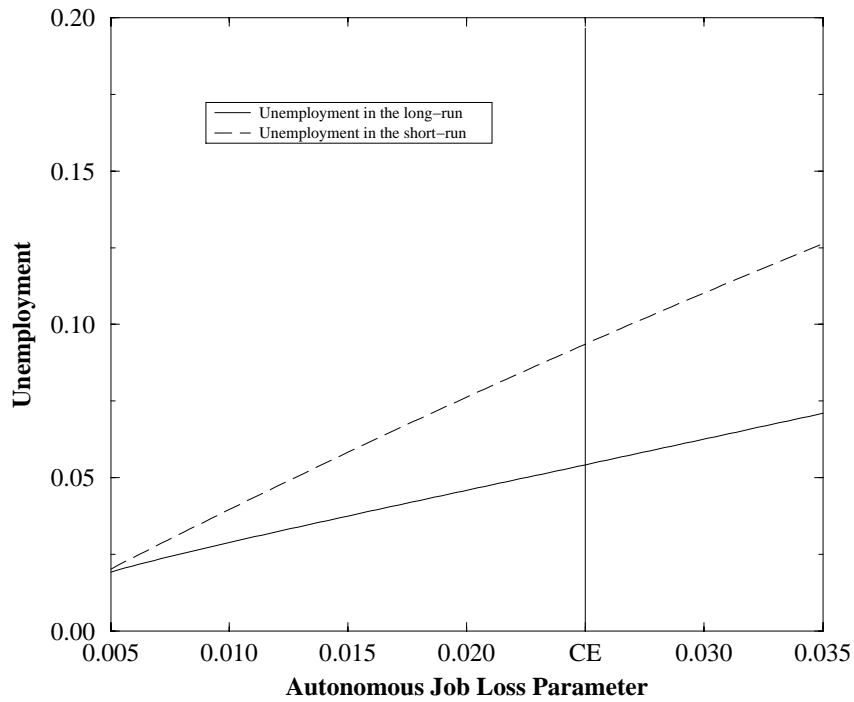
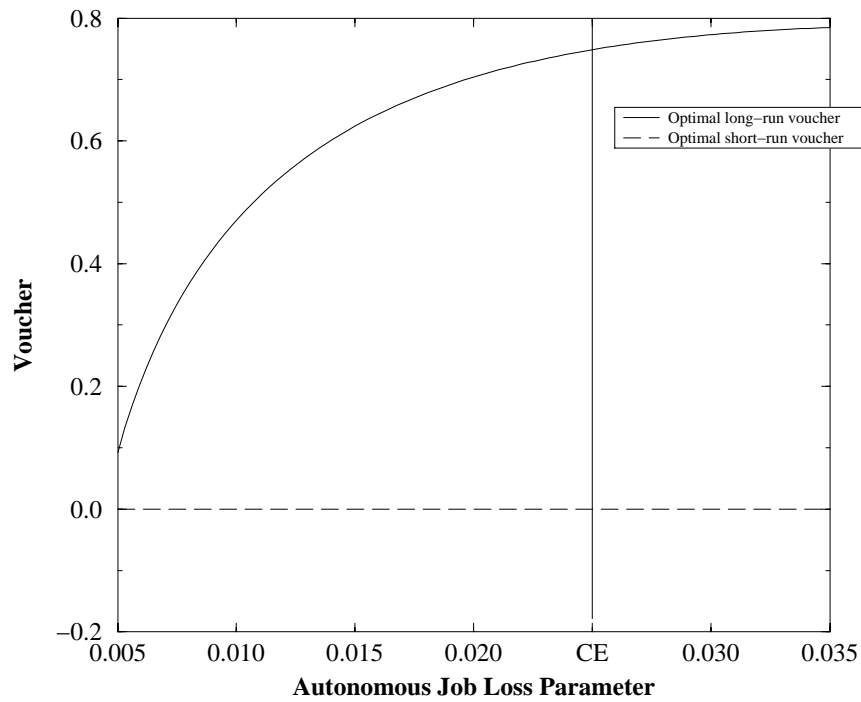


FIGURE 4. The Effects of Autonomous Job Loss

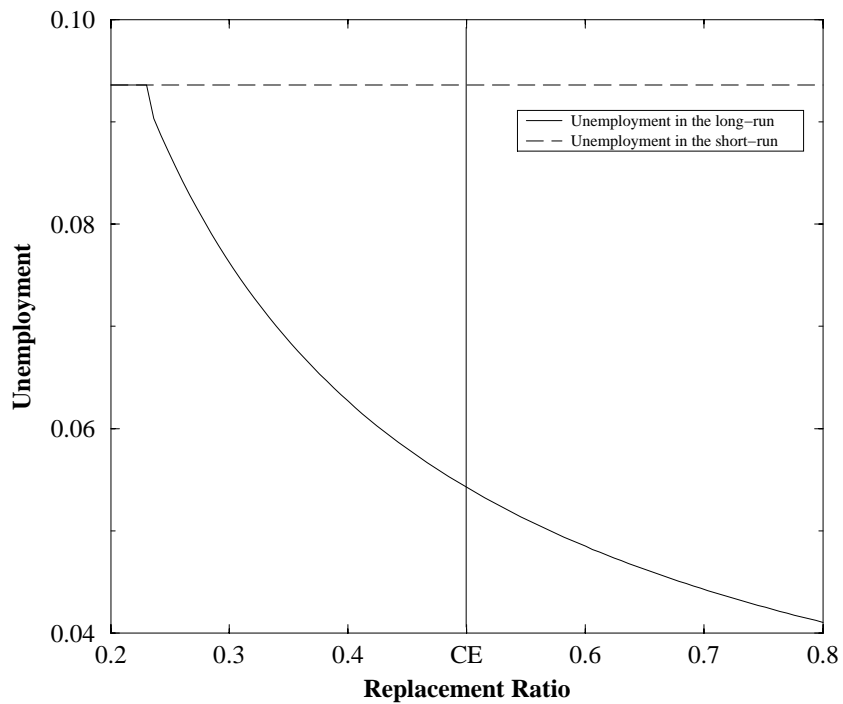
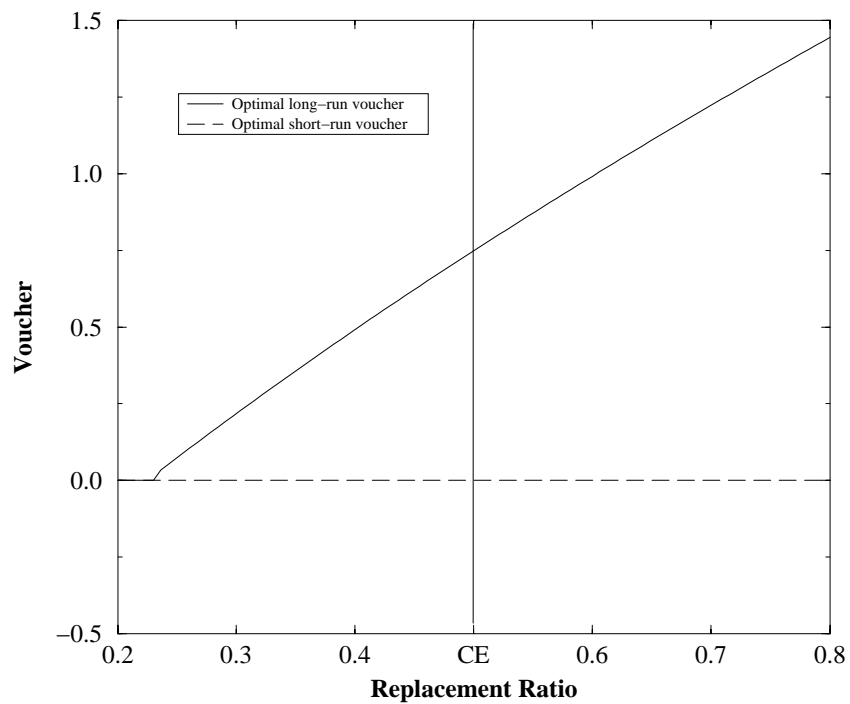


FIGURE 5. The Effects of the Replacement Ratio.

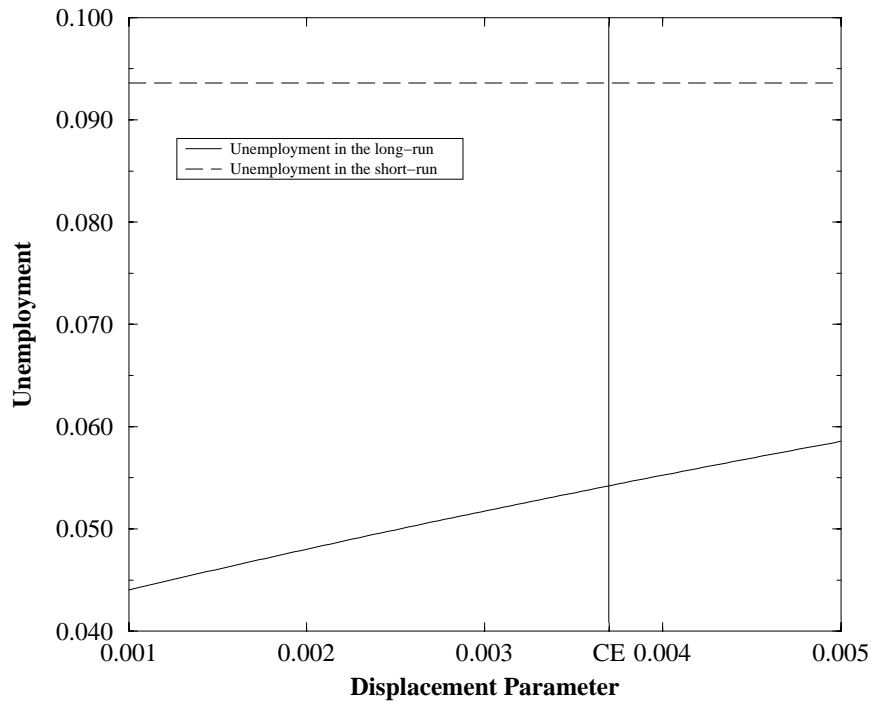
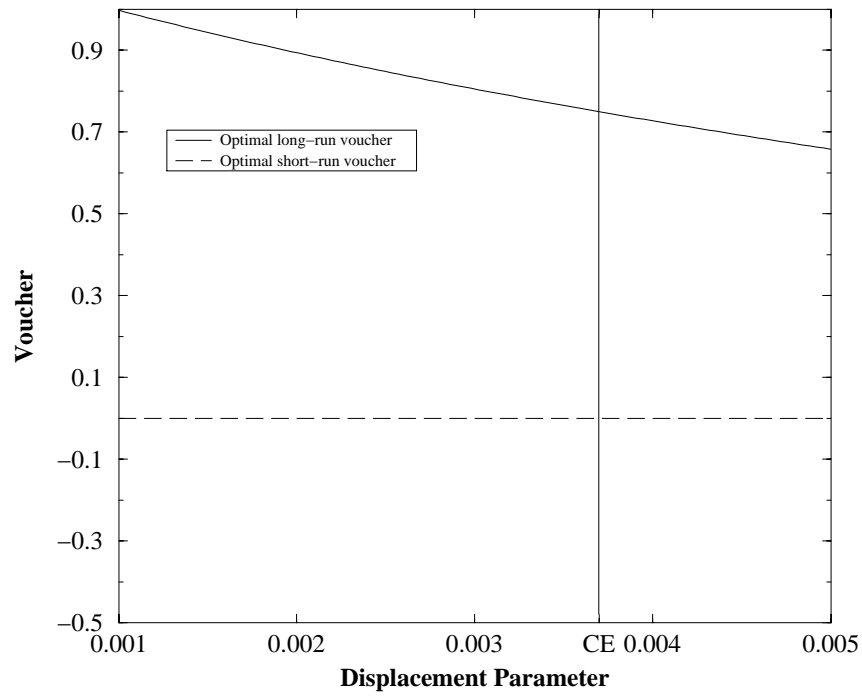


FIGURE 6. The Effects of Displacement.